Explicit Predictions for Illness Statistics

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Abstract
People’s predictions for real-world events have been shown to be well-calibrated to the true environmental statistics (e.g. Griffiths and Tenenbaum 2006). Previous work, however, has focused on predictions for these events by aggregating across observers, making a single estimate for the total duration given a current duration. Here, we focus on assessing predictions for both the mean and form of distributions in the domain of illness duration prediction at the individual level. We assess understanding for both acute illnesses for which people might have experience, as well as chronic conditions for which people are less likely to have knowledge. Our data suggests that for common acute illnesses people can accurately estimate both the mean and form of the distribution. For less common acute illnesses and chronic illnesses, people have a tendency to overestimate the mean duration, but still accurately predict the distribution form.

Keywords: Prediction; Judgment; Health; Cognition

Introduction
Imagine that you have the flu and need to decide whether you will be better in time to travel to a conference this weekend. You are now faced with predicting how long you will be sick. For this inference, you will need to use your knowledge of real-world statistics, including both the mean duration and most likely form of the duration distribution.

People have been shown to make optimal predictions for the duration of many real-world events (Griffiths & Tenenbaum, 2006). In these domains, people’s beliefs about the underlying distribution of quantities (e.g. cake baking times are captured by a bimodal distribution) have been shown to be accurate in the aggregate. These findings have been extended to people’s ability to make predictions for illness duration (Robbins and Hemmer, in revision). People were able to make predictions that were consistent with both the mean and form of illness distributions for common acute illnesses (e.g. common cold and seasonal flu), but systematically overestimated the duration of chronic illnesses (illnesses with which they had significantly less experience). This suggests they had knowledge of the correct form of the underlying illness duration distributions.

One limitation of the procedures used in previous experiments (e.g. Griffiths & Tenenbaum, 2006) is that each participant made only one prediction about a total duration given its current duration. As such, data was aggregated over participants to assess the fit of participant data to the true duration distributions. As Griffiths and Tenenbaum (2006) explain, this gives a guide to peoples’ implicit beliefs about the distributions. As such, these experiments do not allow for an assessment of whether people have knowledge of the correct form of the underlying illness distribution at the individual level. Accordingly, these studies could be illustrating the wisdom of the crowds effect, whereby aggregating over many individual judgments from a group of people leads to a response that is closer to the ground truth than that of a smaller group (Surowiecki, 2004).

To our knowledge, no previous work has assessed the correspondence between people’s beliefs and the statistics of the environment—specifically illness statistics—at the individual level. Therefore, in the current study, we assessed whether people understood the true statistics for the durations of different illnesses by asking them directly what they thought the mean and correct form of illness duration distributions were. This allowed us to evaluate whether people have an internal model for real-world statistics that they can consciously access and use to make predictions.

Understanding illness duration is critical for illness identification. For instance, imagine you have a cough and high fever, and thinking you have the flu you try to estimate how long you will be sick. One thing you will draw on is your understanding of the real-world distribution of durations for different illnesses. If your symptoms begin to fade after three days, this may confirm your suspicion that you have the flu, since this is within the normal distribution for the flu. However, if you are still sick after 10 days, you might begin to believe you have a different illness such as the common cold, because you know that 10 days is reasonable within the distribution of duration for the common cold. This estimation requires an understanding of the entire distribution of illness duration, rather than just the mean or some conditional duration. With only the mean of the distribution, you would not know how much variation in duration is normal, or at which point a particular illness is unlikely given the duration of your symptoms.

Illness further provides an interesting example for prediction because people have different levels of experience for different illnesses—e.g., common illnesses such as the cold, or less common illnesses such as bacterial meningitis. Experience may also differ between acute (e.g. cold) and chronic (e.g. asthma) illnesses. An acute illness is defined as one which can be cured with treatment, while a chronic illness is defined as one that can be managed but not cured. Differing levels of experience between chronic and acute illnesses may influence the accuracy of a person’s prior beliefs, and different priors might be appropriate for different illnesses, given personal experience.

The observer’s prior beliefs play an important role, as optimal predictions are assumed to follow Bayesian principles. Bayes rule gives a principled account of how people should update their prior beliefs given evidence from the world. Each time a person experiences an illness, they
should update their prior probability distributions for the duration of that illness. This would result in illnesses that are experienced more often having very accurate prior distributions. For illnesses that are less commonly experienced, people might adjust their prior beliefs to those of illnesses for which they have more knowledge of the correct form of the distribution, when making inferences. While people might use evidence from other sources when updating their priors, evidence that is personally experienced is better integrated than information acquired in other ways (Sallnas, Rassmus-Grohn, & Sjostrom, 2000).

In this paper, we sought to assess whether the correspondence of people’s beliefs to Bayesian optimal predictions in the aggregate (e.g. Griffiths & Tenenbaum, 2006; Robbins & Hemmer, in revision) extended to estimations for the mean and correct form of illness duration distributions at the individual level. We further sought to apply this approach to a domain with direct implications for real world problems—specifically patient health. In Experiment 1, we simply asked participants to predict the mean duration of each of nine illnesses.

In Experiment 2, we sought to assess whether people could make estimations of the correct form of illness distributions. To do this, we gave participants four distribution options—each fit to the true clinical data for that illness—and asked them to select the distribution form that best described that illness. Because each of the distribution options was fit to the clinical data, consistent selection of the correct distribution would clearly illustrate that there is a correspondence between people’s internal model and the true statistics of the environment. This suggests that they have a cognitive representation of the form of the distribution of durations that they can consciously access.

**Experiment 1: Mean Estimation**

**Methods**

**Participants** Ninety-Nine Mechanical-Turk workers from the United States participated in exchange for $1.

**Materials** We selected nine illnesses—five acute and four chronic (see Table 1)—intended to span a range of durations and familiarity. Familiarity was determined based on prevalence statistics for the number of people diagnosed with that illness each year (see Table 1). Table 1 also includes the source of the clinical data used for the illness duration distributions.

We first needed to determine the mean and correct form of the nine illness distributions. Illness durations have been found to be well modeled by a type of distribution known as a survival function, which includes Gamma, Exponential, and Weibull. The Erlang distribution is a special case of the Gamma distribution, where \( \alpha \) must be an integer, which is often used to model illness duration and illness stages in transmission models of infectious disease, and to infer parameters from clinical data (Krylova & Earn, 2013). For this reason, we assume Erlang is the correct distribution for the nine illnesses. See Figure 1 for the clinical duration distributions for the nine illnesses used in this experiment, with corresponding Erlang distribution fits. The clinical data provides a ground truth for both the mean and form of distributions to compare to participant responses (see Table 1 for clinical data sources).

**Procedure** The procedure was identical to that of Griffiths and Tenenbaum (2006), with the important difference that we did not condition on the current unit of time. As a consequence of the units of time available in the experiment, there may have been an anchoring effect, which is when people are systematically influenced by starting points regardless of whether they are informative (e.g. Chapman & Johnson, 1999). By not providing the units of measure, we eliminate any possible anchoring effect. Our current procedure provides a truer picture of people’s ability to estimate the mean because they are not given a frame of reference. Participants simply made a prediction about the total duration of each of the nine illnesses. The question read: “Given that you meet someone with illness X, what do you think will be the total duration of their illness?” Participants responded by typing in a number and selecting a unit of time from a dropdown menu presented on the computer screen. The experiment was performed using the Qualtrics interface. The order of presentation was randomized.

Participants were also asked to categorize each illness

**Table 1: Sources for Clinical Data (in order of prevalence)**

<table>
<thead>
<tr>
<th>Illness</th>
<th>Source of Clinical Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Prevalence/10,000)</td>
<td></td>
</tr>
</tbody>
</table>

**Acute (in order of prevalence)**

- Bacterial Meningitis (.14) — Kilpi & Anttila (1991)
- Mononucleosis (5) — Cameron et al. (2006)
- Appendicitis (9) — Singh et al. (2014)
- Seasonal Flu (1250) — Kohno et al. (2010)
- Common Cold (2360) — Gwaltney, J. (1967)

**Chronic (in order of prevalence)**

- COPD (4.5) — Shavelle (2009)
- Asthma (800) — American Lung Association (2012)
- Type II Diabetes (860) — http://www.cdc.gov/diabetes/statistics/duration/fig1.htm
- Chronic Heart Disease (1130) — Proudfoot et al. (1983)

![Figure 1: Histograms of clinical data for nine illnesses with best fitting Erlang distributions. Grey bars show the frequency of each illness duration, black lines show the Erlang fit to clinical data. \( \mu \) gives the distribution mean.](http://www.cdc.gov/diabetes/statistics/duration/fig1.htm)
using one of five labels: “Lasts a short time, will go away completely even without treatment”, “Can vary in length, requires immediate treatment, but can be cured”, “Is long term, requires treatment, but can eventually be cured”, “Lasts the rest of a person’s lifetime, treatment can only manage symptoms, it cannot be cured, but does not necessarily cause death”, “Varies in length, treatment can only manage symptoms, cannot be cured, eventually causes death”. Participants were also asked several basic demographic questions (e.g. age and experience with the nine illnesses) which are not analyzed here.

**Results**

Given that participants could respond with any unit of time, we first normalized participant responses to the unit of time for the clinical distributions. Responses were then filtered for outliers. Data was excluded in the following way: unreasonably large responses (defined as those 3 standard deviations greater than the mean response for a given illness) and participants who had more than two data points excluded based on the above criteria. The responses analyzed were 85 for appendicitis, 90 for the seasonal flu, 90 for the common cold, 87 for bacterial meningitis, 77 for mononucleosis, 90 for COPD, 90 for chronic heart disease, 90 for type II diabetes, and 90 for asthma.

First, we examined people’s ability to characterize the durations of acute and chronic illnesses. Chronic illnesses are lifelong, which is a critical difference from acute illnesses which are curable. To determine whether participants had basic knowledge of the illnesses they were making estimations about, we examined their responses to questions asking to characterize each illness. For the common acute illnesses—common cold and seasonal flu—92% of participants correctly responded that the illnesses were short term and curable. For the less common acute illnesses—appendicitis and bacterial meningitis—81% and 66% of participants respectively labeled these illnesses as short term. For the four chronic illnesses 74%-84% of participants correctly responded that these illnesses were lifelong. This clearly shows that people understand the chronicity of the chronic and common acute illnesses.

We first evaluated the accuracy of participant’s mean responses (see Table 2). A qualitative evaluation of the data illustrates that participant responses were close to the true mean for more prevalent acute illnesses (i.e. common cold and seasonal flu), and that participants overestimated the duration of chronic illnesses, similar to the pattern found by Robbins & Hemmer (in revision).

In order to evaluate whether participant responses were accurate relative to the true mean of the empirical illness distributions, we used a two one-sided t-test approach (e.g. Limentani et al., 2005). We used this approach as it allows us to test for practical equivalence (e.g. Rogers, Howard, & Vessey, 1993). A one-sample t-test might find a significant difference between a population mean of seven days and a participant response mean of eight days. While this difference is significant, it places too rigid a standard for our purposes, leading to an inaccurate conclusion that participants do not understand the mean of that illness. For this reason, we set a criterion considering accuracy to be within one standard deviation of the mean of the empirical illness distributions (standard deviations for each illness are displayed in Figure 2). We then conducted a t-test on either end of this threshold to determine if participant responses were significantly greater than the lower threshold, and significantly less than the upper threshold.

We found that for mononucleosis and the common cold, Table 2: *True and estimated illness durations*

<table>
<thead>
<tr>
<th>Illness</th>
<th>True Duration</th>
<th>Participant Response</th>
<th>% using unit of time (correct unit is <strong>bolded</strong>)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hours</td>
<td>Days</td>
<td>Weeks</td>
</tr>
<tr>
<td><strong>Acute</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appendicitis</td>
<td>39 hours</td>
<td>471.6(SD=969.5) hours</td>
<td><strong>8.4</strong></td>
</tr>
<tr>
<td>Seasonal Flu</td>
<td>3.9 days</td>
<td>8.9(SD=4.5) days</td>
<td>2.1</td>
</tr>
<tr>
<td>Common Cold</td>
<td>5.1 days</td>
<td>6.3(SD=3.2) days</td>
<td>1</td>
</tr>
<tr>
<td>Bacterial Meningitis</td>
<td>5.5 days</td>
<td>37.3(SD=44.0) days</td>
<td>2</td>
</tr>
<tr>
<td>Mononucleosis</td>
<td>10 weeks</td>
<td>9.3(SD=13.0) weeks</td>
<td>1</td>
</tr>
<tr>
<td><strong>Chronic</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COPD*</td>
<td>6 years</td>
<td>36.6(SD=22.0) years</td>
<td>0</td>
</tr>
<tr>
<td>Type II Diabetes</td>
<td>12 years</td>
<td>36.0(SD=22.5) years</td>
<td>0</td>
</tr>
<tr>
<td>Chronic Heart Disease</td>
<td>13 years</td>
<td>26.4(SD=20.0) years</td>
<td>0</td>
</tr>
<tr>
<td>Asthma</td>
<td>15 years</td>
<td>42.5(SD=25.7) years</td>
<td>4.2</td>
</tr>
</tbody>
</table>

* COPD stands for Chronic Obstructive Pulmonary Disease

Figure 2: Red bars show the percentage of participants that were X number of standard deviations from the mean. Positive numbers indicate estimations above the mean, and negative numbers indicate estimations below the mean.
responses were within the one standard deviation of the true mean—meaning the estimates were practically equivalent to the true mean (upper threshold: Mononucleosis: t(76)=−6.1, p<.01, Common cold: t(89)=−6.9, p<0; lower threshold: Mononucleosis: t(76)=5.1, p<.01, Common cold: t(89)=13.4, p<.01). For the other seven illnesses, responses were found to be greater than the lower end of the threshold, but not less than the higher end of the threshold, suggesting a pattern of overestimation, (Appendicitis: t(84)=4.3, p<.01, Seasonal flu: t(89)=13.0, p<.01, Bacterial meningitis: t(86)=7.1, p<.01, COPD: t(89)=14.9, p<.01, Type II diabetes: t(89)=20.4, p<.01, Chronic heart disease: t(89)=20.0, p<.01, Asthma: t(89)=13.2, p<.01).

Given that participants were not within the one standard deviation threshold for seven illnesses, we wanted to further examine how misaligned they were for each illness. Therefore, we calculated the percentage of participants at each standard deviation from the mean (see Figure 2). For the common cold and mononucleosis, the majority of participants (approx. 80%) were within one standard deviation, as illustrated in the TOST. For the seasonal flu more than 70% of participants were within four standard deviations of the mean, which may seem like a large deviation from the correct response, however it is also important to note that the standard deviations varied greatly between illnesses. For the seasonal flu, the standard deviation was only 1.73 days, meaning that more than 70% of participants responded within 6.8 days of the true mean. Conversely, for the least prevalent acute illnesses, appendicitis and bacterial meningitis, only 34% and 38% of participants respectively were within four standard deviations of the true mean, with some participants being up to 80 standard deviations away (for appendicitis this corresponded to 1416 hours or 59 days). This illustrates that participants had lower agreement, and less accurate mean estimations for these illnesses.

For the chronic conditions, fewer participants were within four standard deviations of the mean, with 31% for COPD, 100% for type II diabetes, 61% for chronic heart disease, and 47% for asthma. Participant responses were all within four standard deviations of the mean for type II diabetes because the standard deviation is 24 years.

We then examined whether the absence of a time anchor influenced the unit of time participants used to respond (see Table 2). For the acute illnesses, multiple units of time can be used to express the same value; i.e., a one week long illness can be characterized as seven days or one week. For seasonal flu and common cold, more than 80% of participants responded with either the clinical (days) or the adjacent and reasonable (weeks) unit of time. For mononucleosis, approximately 66% of participants used the clinical or adjacent unit of time. For the least prevalent acute illnesses—appendicitis and bacterial meningitis—participants used the clinical or adjacent unit of time only 40% and 55% of the time. For the four chronic illnesses, 92% to 95% of participants chose the clinical unit of time. The results suggest that participants could reliably use the clinical unit of time when estimating durations of prevalent acute illnesses and chronic illnesses.

**Experiment 2: Distribution Form Estimation**

**Methods**

**Participants** Forty Mechanical-Turk workers from the United States participated in exchange for $2. The participants had not participated in Experiment 1.

**Materials** The same nine illnesses from Experiment 1 were used. We selected four distributions as response options in the distributional form task: Erlang, Gaussian (a.k.a. Normal), Uniform, and Bimodal. These distributions were chosen as they can reasonably describe illness durations. The Erlang, which was always the correct answer, was chosen because illness distributions have been found to be well modeled by this distribution and provide a good fit for all the clinical distributions. Normal was chosen because the bell-curve is ubiquitous, and in some cases is very close to the Erlang distribution. This allows us to evaluate how well participants can discriminate very similar distributions. Bimodal was chosen because for chronic illnesses it might be reasonable to assume that there is one group of people who die immediately, and another group that lives with the illness for a longer time. Lastly, uniform was chosen because simple Bayesian prediction models assume a single uninformative (or uniform) prior (e.g. Gott, 1993). Selecting the uniform form of the distribution might suggest observers using a heuristic insensitive to prior beliefs.

Distributions were presented to participants as histograms of the average total duration of an illness. For each illness, the presented histograms were created by producing the best fit to the true clinical data for that illness for each of the four distributions. In this way, participants’ choice of distribution

**Figure 3: Screenshot of experimental interface for sample question (seasonal flu). Distribution types, top left to bottom right, are: Uniform, Normal, Erlang, and Bimodal.**
would be based solely on distribution form. The histograms were presented with descriptive captions. The captions for each distribution form were consistent for all illnesses. Captions described several critical points on the graph using frequencies out of 100 (see Figure 3). The descriptions for each distribution form were matched to illustrate the same number of points on the histogram. Four naïve raters evaluated the relationship between the descriptors and the histograms and in all cases found them to be well-matched and easily understood. The experiment was presented using the Qualtrics interface.

**Procedure** Participants were first shown instructions on how to read graphs in our task. They then completed a training task, with two training sessions of four trials each. For each trial, participants were shown one histogram (illustrating one of the four distributions types used throughout this experiment) and asked to match it to one of four captions. The training trials were designed to illustrate duration without referencing illnesses. One set depicted the amount of time it takes for a person to turn into a zombie after being bitten, and the second set depicted the number of licks it takes to get to the center of a tootsie pop.

After the training task, participants were asked to choose the appropriate histogram from the four distribution options for each of the nine illnesses (presented one at a time) by selecting it with a radio button. Both question and choice order were randomized.

**Results**

Data were excluded if participants answered two or more questions incorrectly in each of the two four trial training-sets. This removed two participants’ data from analysis.

First, we assessed the proportion of trials for which participants chose the clinical distribution (Erlang). Participants chose Erlang 42% of the time, which was significantly greater than chance (25%), based on a one-sided Binomial test ($p<.01$). It was also chosen significantly more often than any of the other distributions: Normal $X^2(1,N=342)=11.8$, $p<.01$, Uniform $X^2(1,N=342)=93.9$, $p<.01$, and Bimodal $X^2(1,N=342)=48.0$, $p<.01$.

While participants selected Erlang with the greatest frequency overall, we were further interested in how frequently they chose the correct response for each individual illness. We performed a one-sided Binomial test and found that for six of nine illnesses, participants performed better than chance (i.e. significantly more than 25% of participants chose the Erlang distribution): seasonal flu (53%, $p<.01$), common cold (50%, $p<.01$), bacterial meningitis (42%, $p=.016$), mononucleosis (42%, $p=.016$), COPD (45%, $p<.01$), type II diabetes (47%, $p<.01$). Participants did not select any of the other distributions at a level higher than chance. See Figure 4 for the proportion of participants that chose each distribution option for the nine illnesses.

Lastly, we performed a chi squared test to determine whether participants selected the Erlang distribution significantly more often than the other distribution choices.

Participants chose Erlang more often than Uniform for eight out of nine illnesses: seasonal flu $X^2(1,N=38)=18.0$, $p<.01$, common cold $X^2(1,N=38)=19.0$, $p<.01$, bacterial meningitis $X^2(1,N=38)=9.8$, $p<.01$, mononucleosis $X^2(1,N=38)=17.0$, $p<.01$, COPD $X^2(1,N=38)=13.3$, $p<.01$, chronic heart disease $X^2(1,N=38)=7.9$, $p<.01$, type II diabetes $X^2(1,N=38)=8.8$, $p<.01$, and asthma $X^2(1,N=38)=4.7$, $p=.03$.

Erlang was chosen significantly more than Bimodal for five of nine illnesses: seasonal flu $X^2(1,N=38)=9.7$, $p<.01$, common cold $X^2(1,N=38)=4.5$, $p=.03$, bacterial meningitis $X^2(1,N=38)=8.0$, $p<.01$, COPD $X^2(1,N=38)=9.2$, $p<.01$, and type II diabetes $X^2(1,N=38)=23.6$, $p<.01$.

Participants chose Erlang significantly more than Normal for two out of nine illnesses: seasonal flu $X^2(1,N=38)=8.1$, $p<.01$, and common cold $X^2(1,N=38)=8.4$, $p<.01$. As shown above, Erlang was chosen significantly more often than any other distribution for both seasonal flu and common cold.

![Figure 4: Red bars show the percentage of participants that chose a distribution choice.](image)

**General Discussion**

We evaluated people’s ability to estimate the mean and correct form of duration distributions at the individual level within the domain of health. Examining people’s representations of illness duration statistics is important, because it allows us to understand the correspondence between people’s beliefs and the statistics of the environment—in this case—illness statistics. In addition, these experiments shed light on people’s internal representations of real world statistics.

Our most interesting finding is that participants appeared to have knowledge of the correct form of the underlying illness distribution, choosing the assumed clinical distribution (Erlang) more frequently than any other distribution. When broken down by illness, they chose the clinical distribution more frequently for the most prevalent acute illnesses: seasonal flu and common cold.

While participants often inferred the form to be the normal distribution, this may be explained by the similarity of many of the normal fits to the Erlang fits. This occurred because the normal distributions were truncated by a lower duration bound of zero. We deliberately included the Normal distribution because of the potential confusability with the clinical distribution. As such, the fact that participants still chose the clinical distribution as the correct form overall, suggests they have strong beliefs about the
form of illness duration distributions and that these correspond to the environmental statistics. It is important to note that research has illustrated that people often fail at graphical interpretation (e.g., Gerteis et al., 2007), which makes participant performance in this task impressive. When examining participants’ estimates for the mean, we found that for more prevalent acute illnesses (i.e., common cold and seasonal flu), they were able to accurately estimate the mean duration. We also found a pattern of overestimation for chronic illnesses and less-prevalent acute illnesses which was similar to the pattern of overestimation found by Robbins and Hemmer (in revision).

The pattern of overestimation for chronic illnesses might be explained by people applying a probabilistic model of life expectancy to their understanding of the distribution form for illness durations. Because they have little experience with chronic illnesses, and they understand that chronic illnesses are life-long, their overestimation might be due to a strategy of applying parameters from the true distribution of lifespans (adjusted slightly to account for decreased life-expectancy with a chronic illness) to their knowledge that illnesses follow the form of an Erlang distribution. Their ability to select the appropriate distribution form for these illnesses suggests that they can use knowledge of the form of other illness distributions even if they do not have enough experience to set the parameters accurately. This overestimation might also be adaptive in terms of planning for the future. For chronic illnesses, it may be safer to assume a longer duration to plan sufficiently for the future, i.e., retirement savings.

A logical next step for this work would be to ask participants to independently generate distributions, rather than asking them to select from a limited number of options. Goldstein & Rothschild (2014) have shown that participants can generate these distributions when presented with data, which suggests that this method could be used to evaluate peoples’ internal representations of real-world statistics.

Our results illustrate that people hold accurate representations for both the form and mean of duration distributions of prevalent acute illnesses. Significantly, the most prevalent acute illnesses—the common cold and seasonal flu—are also the ones for which participants consistently demonstrate knowledge of the correct distribution form, and accurately predict the mean at the individual level. This suggests a prior belief that is better calibrated to the true environmental statistics for illnesses participants have experience with. Taken together, the data suggests that people have an internal representation of illness statistics that they can consciously access—indicating that people can not only combine illness experiences with rational statistical updating, but also have accurate knowledge of these prior distributions.

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**References**


