A Common Neural Component for Finger Gnosis and Magnitude Comparison

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Abstract

Finger gnosis (the ability to identify which finger has been touched) and magnitude comparison (the ability to determine which of two numbers is larger) are surprisingly correlated. We present a spiking neuron model of a common component that could be used in both tasks: an array of pointers. We show that if the model’s single tuned parameter is set to match human accuracy performance in one task, then it also matches on the other task (with the exception of one data point). This provides a novel explanation of the relation, and proposes a common component that could be used across cognitive tasks.

Keywords: finger gnosis; magnitude comparison; spiking neurons; neural engineering framework, numerical cognition

Introduction

Finger gnosis, the ability to differentiate which finger has been touched, in absence of visual feedback, is related to math performance (Fayol, Barrouillet & Marinthe, 1998; Noël, 2005; Penner-Wilger et al., 2007, 2009, 2014, 2015). Finger gnosis is commonly measured using a finger localization task (Baron, 2004; Noël, 2005), wherein the participant’s hand is occluded from their view while a finger, or two fingers, are touched. The participant is then asked to indicate the touched finger(s). Performance is measured in terms of number of fingers correctly identified.

Finger gnosis ability predicts performance on a variety of math measures in children, both concurrently and longitudinally (β’s range from .22 to .36; Fayol et al., 1998; Noël, 2005; Penner-Wilger et al., 2007, 2009). Finger gnosis ability also predicts performance on a variety of math measures in adults (β’s range from .21 to .30; Penner-Wilger et al., 2014, 2015). The relation between finger gnosis and math skill is reproduceable across labs, different samples, age groups, and measures of math skill, despite controlling for many other variables (e.g., visuo-spatial working memory, finger agility, processing speed, and non-verbal IQ).

The relation between finger gnosis and math skill is partially mediated by symbolic number comparison performance (Penner-Wilger et al., 2009, in prep.). In symbolic number comparison tasks, participants are shown two digits (e.g., 2 3) and asked to indicate which number is more (or in some variants asked to compare a target digit to a standard). One robust finding in number comparison is the distance effect – performance is faster and more accurate when numbers are father apart in magnitude (e.g., 2 7) than when they are closer together (e.g., 2 3; Moyer & Landauer, 1967). The distance effect is proposed to reflect mapping between numerals and their associated magnitude, with greater distance effects reflecting noisier mappings (Dehaene, Dehaene-Lambertz & Cohen, 1998; cf. Lyons, Nuerk & Ansari, 2015). Children who perform better in finger gnosis, reflecting a more precise finger representation, also demonstrate smaller distance effects in number comparison, reflecting a more precise number representation (Penner-Wilger et al., 2009).

Why are finger gnosis and math performance, specifically a task indexing the precision/strength of number representations, related? On the redeployment view (Penner-Wilger & Anderson, 2008, 2013), the relation between finger gnosis and number representation arises because the two tasks use overlapping neural substrates. On this view, the relation is an example of neural reuse, the use of local regions of the brain to support multiple tasks across domains (Anderson, 2010, 2014). Neural reuse is a dynamic process, impacting the functional organization of the brain across both evolutionary and developmental time, whereby individual regions of the brain contribute to multiple high-level uses (e.g., finger representation and number representation). There are two forms of neural reuse: redeployment and neuromodulation. In redeployment, the same brain region supports multiple uses, across evolutionary and/or developmental time, while maintaining the same operation (Anderson, 2014). In neuromodulation, the same brain region supports multiple uses, at any given point in developmental time, without maintaining the same operation – its operation is modulated as a result of internal or external variables (Anderson, 2014; Bargmann, 2012; Marder, 2012). The redeployment view posits that the behavioural link between finger and number representations is at least partially explained by neural reuse, and that the specific type of neural reuse involved is redeployment. Thus, one (or more) local brain regions, over evolutionary
and/or developmental time, has come to perform the same operation in support of both uses.

In support of the redeployment view, regions associated with finger gnosia are activated during tasks requiring the representation of number (Andres, Michaux & Pesenti, 2012; Dehaene et al., 1996; Zago et al., 2001), rTMS and direct cortical stimulation disrupt both finger gnosia and tasks requiring the representation of number (Rusconi, Walsh, & Butterworth, 2005; Roux et al., 2003), and there is interference between tasks involving finger gnosia and tasks requiring the representation of number (Brozzoli et al., 2008). Zago et al. (2001) pinpointed a region of overlap between finger and number representation in the left-precentral gyrus (-42, 0, 38). Penner-Wilger and Anderson (2011) conducted a meta-analysis of imaging data to determine the full complement of tasks, across domains, that this ROI was implicated in, with the goal of identifying common requirements of tasks/uses to guide structure-function mapping. In addition to number and finger representation tasks, the ROI was implicated in generation, inhibition and order tasks. Common requirements across these uses were identified, including ordered storage and mapping, and a candidate working that could implement both these requirements was proposed—an array of pointers. An array is an ordered group, meeting the requirements for ordered storage, and a pointer is a data structure that designates a memory location and can indicate different data types. Thus, an array of pointers allows for storage and access of ordered elements, which are able to point to—or index—representations or locations in memory, allowing for mapping between different representational forms.

The neural overlap between finger and number representation could reflect redeployment, wherein the brain region is reused in both tasks while retaining the same operation. Alternatively, the overlap could reflect neuromodulation, wherein the operation of the region is modulated. In the current paper, we use computational modelling as a means of demonstrating whether the same proposed working—an array of pointers—could contribute to both number and finger representation. The goals of the current research are to evaluate the redeployment view and proposed shared working by (1) providing an in-principal demonstration that the same working could contribute to both uses, (2) determining the psychological plausibility of the model by comparing it to human performance on finger gnosia and number comparison tasks, and (3) differentiating between support for redeployment (same ROI, same working) over neuromodulation (same ROI, but different working).

**Common Component: A Cognitive Pointer**

The core theoretical claim here is that both finger gnosia and magnitude comparison could plausibly make use of a neural system that is able to store a list of items, and each of those items can be used to indicate other information. For example, these items could mean a particular number (e.g. ONE or THREE) or they could mean any other known concept. For the purposes of this paper, we choose these vectors randomly, but we could use other vector-based representation methods such as LSA or word2vec.

To be explicit about what we mean by such a system, let us define it mathematically. First, we need a (small) set of numerical values which are our “pointers”: \( p_1, p_2, p_3, p_4, \) and \( p_5 \). For the purposes of this paper, we keep the size of this set to 5 (the number of fingers on a hand). Each of these pointers is a numerical vector, and different values can have different meanings. For example, there could be one value that means the number ONE, with other values meaning other concepts like DOG.

In the absence of input, these pointers should not change their value. However, we also need some way of changing their value when needed. For this, we need two things: a new input value \( x \) and a way to indicate which pointer should be set to the new value. This input control is a mask \( m \) and it is a list of values indicating which pointer should be set. For example, if \( m=[0,1,0,0,0] \), then the input \( x \) will be set to the second pointer \( p_2 \).

Mathematically, we can write this as follows, where \( i \) indexes the different pointers:

\[
\begin{align*}
p_i & \leftarrow \begin{cases} 
p_i & \text{if } m_i = 0 \\
x & \text{if } m_i = 1 
\end{cases}
\end{align*}
\]

We postulate that the two tasks use this component as follows. For the finger gnosia task, consider what happens if two fingers are touched, the index finger and the ring finger. We can treat each pointer as a separate finger, and load in a vector that means TOUCHED into the correct pointers by setting \( x=\text{TOUCHED} \) and \( m=[0,1,0,1,0] \).

For the magnitude comparison task, we load the first value into the first pointer and the second value in the second pointer. For the case of comparing 5 and 7 this means setting \( x=\text{FIVE} \) and \( m=[1,0,0,0,0] \), and afterwards setting \( x=\text{SEVEN} \) and \( m=[0,1,0,0,0] \). Over time, this process proceeds stepwise as follows, and maintains its state as shown:

**Finger Gnosis Task**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( m )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
<th>( p_4 )</th>
<th>( p_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>--</td>
<td>00000</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>TOUCHED</td>
<td>01010</td>
<td>TOUCHED</td>
<td>TOUCHED</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>--</td>
<td>00000</td>
<td>TOUCHED</td>
<td>TOUCHED</td>
<td>--</td>
<td>--</td>
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</tr>
</tbody>
</table>

**Magnitude Comparison Task**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( m )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
<th>( p_4 )</th>
<th>( p_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>--</td>
<td>00000</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>FIVE</td>
<td>10000</td>
<td>FIVE</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>SEVEN</td>
<td>01000</td>
<td>FIVE</td>
<td>SEVEN</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>--</td>
<td>00000</td>
<td>FIVE</td>
<td>SEVEN</td>
<td>--</td>
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</tr>
</tbody>
</table>

**Neural Implementation**

While the above algorithm gives us a conceptual understanding of this array of cognitive pointers, we also want to determine how neurons could implement such an algorithm. By examining this neural mechanism, we can...
gain insights into how accurate it would be in different conditions, and hopefully gain insight into individual differences and cognitive deficits.

For our neurons, we use standard leaky-integrate-and-fire (LIF) neurons. These increase in voltage given their input, and emit a spike and reset when the voltage reaches a threshold. These spikes are transmitted to all neurons that the spiking neuron is connected to, with synaptic weights controlling how much current is added to (or subtracted from) the target neuron each time a spike occurs. Each connection also has a post-synaptic time constant that controls the time it takes for a spike's effect to decay away.

Within each group of 400 neurons, each individual neuron has a randomly chosen preferred vector. That is, each neuron will have some particular \( x \) value for which it fires the fastest. This is a generalization of the standard preferred direction vectors observed throughout cortex (e.g. Georgopoulos et al., 1986).

To generate the actual connections between neurons, the NEF uses least-squares minimization to directly solve for the optimal synaptic connection weights that will do the best job of transferring a value \( x \) from one group to the next. This same process is used to generate the recurrent connections for the pointers.

The synaptic time constants were set to 10ms for the feed-forward connections (based on the fast AMPA synapses found in cortex) and 100ms for the recurrent connections (based on the slower NMDA synapses found in recurrent connections in cortex).

It should be noted that there is nothing in the model so far that is fit to a particular task. The optimization of the connection weights is over all possible \( x \) values, not the particular \( x \) values that mean ONE or TWO or TOUCHED in the magnitude comparison and finger gnosis tasks themselves. This is meant to be a generic component, not one that is specialized for exactly these tasks.

To actually create this network, we use the Neural Engineering Framework (Eliasmith & Anderson, 2003) and the software toolkit Nengo (Bekolay et al., 2014). In this approach, we assume that \( x \) is a vector of some dimensionality that is smaller than the number of neurons in a group. This means that there is redundancy in the neural code, and the value \( x \) is distributed across the neural population. Here, for simplicity, we assume \( x \) is an 8-dimensional vector. Previous work (Crawford, Gingerich & Eliasmith, 2013) has shown that 512-dimensions should be sufficient for high-level reasoning applications, but that is not needed for the tasks considered here.

Figure 1 shows the basic approach used to implement this functionality. The groups of neurons on the right store the individual pointer values \( p_1, p_2 \), etc. They are recurrently connected such that they will have stable firing patterns over time (i.e. whatever pattern of firing is present right now will cause a similar firing pattern in the near future).

The connections from \( x \) to the channels and from the channels to the pointers are all set such that the neurons simply pass along the value without altering it. That is, if we input a particular value \( x \), this will cause a particular (and unique) firing pattern in channel 1 and channel 2. These in turn will cause particular firing patterns in pointer 1 and pointer 2. If \( x \) is removed (i.e. set to zero), then the pointer patterns will stay as they were. Thus, they implement a memory of previously presented patterns.

However, we also want to be able to selectively set one or the other pointer. For this reason, we also include the \( m \) (mask) input. This can selectively inhibit the channel neurons. If these are inhibited, then they do not fire, and so do not affect the pointer neurons. So, if we want to set the value in pointer 2 only (and not change whatever is stored in pointer 1), then we inhibit channel 1 when inputting \( x \).

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Figure 2 shows the neural activity when this array of pointers is used to store two numbers. Initially, both pointer 1 and pointer 2 are firing with some random background firing rate. At t=0.2s, we set the input \( x \) to be the vector for FIVE (randomly chosen) and set the mask such that channel 1 is the only group not being inhibited. This drives the neurons in pointer 1 to also fire with the pattern for FIVE. At t=0.4s, we change the input to SEVEN and change the mask so that channel 2 is not inhibited. This drives pointer 2 to represent SEVEN. Importantly, after \( x \) is removed, the neurons in pointer 1 and pointer 2 retain their firing pattern.

Figure 1: Neural implementation of an array of pointers. Only two pointers are shown.
While the system described above behaves as desired, note that it is not perfect. The neurons in channel 1 and channel 2 are not perfectly inhibited. Also, the neurons in pointer 1 and pointer 2 do not perfectly maintain exactly the desired firing pattern. This is as expected, as neurons only approximate the desired functions. We can now test whether the resulting model can still perform the two tasks, and whether the errors made by the model due to these imperfections are comparable to the errors made by people.

**Task 1: Magnitude Comparison**

The first task believed to make use of this component is magnitude comparison. Two single-digit numbers are presented, and the system must decide which is larger.

To implement this task neurally, we add two new neural groups. First, a comparison group, which takes as input the vectors in the first two pointer populations. This means that the comparison group has a 16-dimensional input, with the value from $p_1$ as the first 8 dimensions and the value from $p_2$ as the second 8 dimensions. Second, we have an accumulator. This takes as input a single number which should be positive if the first number is larger, and should be negative if the second number is larger. This is recurrently connected to itself, so that even for small inputs, it will eventually build up until it reaches a threshold, making it a standard accumulate-to-threshold decision-making system.

To evaluate this model, we collected human participant data from 88 undergraduate students at King’s University College who received course-credit for their participation (age: $M=21.28$ years, $SD=3.8$ years; 64 female). Two single digit numbers (ranging from 1 to 9) were presented simultaneously on an iPad screen. Participants were asked to choose the numerically larger number as fast as they could without making any errors. Stimuli remained on the screen for 7800ms or until the participant made a choice, and the time between trials was 1000ms. Participants performed a total of 72 trials. Dependent measures were reaction time and percent error.

**Task 2: Finger Gnosis**

In the finger gnosis task, two fingers are touched on the participant’s hand while that hand is occluded from their view. They must then report which fingers were touched.

To implement this task, we use the same array of pointers, but connect it to a different set of neurons, as depicted in Figure 5. The first group of neurons takes the input from all the pointers and combines them together as one vector. The second group stores the reported answer. As with the previous task, we use Nengo to find the connection weights.
that best approximate the function between the combination neurons and the answer neurons. In this case, however, rather than determining which value is larger, here we do not need to perform any complex operation as we just need to extract the information that is already encoded in the neurons. Thus, here we use Nengo to approximate the identity function, where the output is the same as the input.

![Diagram of the finger gnosis model](image)

Figure 5: The finger gnosis model. Only 2 pointers are shown, but the full model uses 5 pointers.

Importantly, if this same array of pointers is to be used in two different tasks, a flexible neural routing system would be needed, so that the output of the pointer array can be sent to this combination system when doing the finger gnosis task, and sent to the comparison system when doing magnitude comparison. We have previously shown how to implement such a routing system using a model of the cortex-basal ganglia-thalamus loop (Stewart, Choo, & Eliasmith, 2010), and so do not consider that here.

To evaluate this model, we used the same 88 undergraduates as for the first task. Participants first performed the magnitude comparison task, followed by the finger gnosis task as part of a larger study. As shown in Figure 6, a repeated-measures ANOVA revealed that mean percent error differed significantly between distances, $F(3, 261) = 6.88, p < .01$.

Figure 6 also shows the model performance. Importantly, no parameters were tuned to achieve this result. We used $s_{inhibition} = 0.875$, as that was the best fit value in the first task, and all other parameters were left as they were. The model is statistically significantly different at a distance of 1, but does not statistically differ for distances 2, 3, and 4.

![Figure 6: Results from participants and model for the finger gnosis task. Standard errors of the mean are shown.](image)

Since the only tuned parameter in the model is $s_{inhibition}$ we also examined how the model's performance changes on the two tasks as this parameter is varied (Figure 7). From this, we note that the error rates on these two tasks change drastically, given small changes in this parameter. This indicates a strong connection between the model's performance on one task and on the other. The fact that a similar parameter value is needed in each task in order to fit the human data lends support to the idea that there is a shared working that is redeployed for these two tasks.

![Figure 7: Effects of changing s_{inhibition} in both tasks.](image)

Conclusions

On the redeployment view (Penner-Wilger & Anderson, 2008, 2013), finger gnosis and math ability are linked because at least one local brain region, over evolutionary and/or developmental time, has come to perform the same operation in support of both finger and number representation. The goal of the current research was to evaluate the redeployment view and the proposed shared operation – an array of pointers (Penner-Wilger & Anderson, 2011). To this end, we built a computational model to perform both the standard finger gnosis and number comparison tasks. We then compared the performance of this model to human performance data (RT and accuracy) and showed a close match on both tasks with one parameter ($s_{inhibition}$) tuned to a common value.

First, our work provides an in-principal demonstration that the same working – an array of pointers – could contribute to multiple uses, as the same system successfully performed two different tasks. Our previous meta-analysis (Penner-Wilger & Anderson, 2011) also indicates this region may be involved in a variety of other tasks, which we intend to include in future research.

Second, given that the model could successfully perform both tasks using the same operation, and that the model performance mirrored that of human participants, it is a psychologically plausible explanation, which lends support for the view that the observed neural overlap between finger and number representation reflects redeployment (same ROI, same working) rather than neuromodulation (same ROI, different working). It follows that damage to the ROI should impact performance on both finger gnosis and number comparison tasks. We are currently testing this in our computational model and it could be tested in human participants using rTMS applied to our ROI in the left precentral gyrus. Previous work using rTMS applied to the left angular gyrus has already been shown to disrupt performance on both tasks (Rusconi et al., 2005).

Third, by offering another concrete instance of the reuse of a basic operation in a high-level, abstract cognitive task, the model does not just bolster the neural reuse framework, but also serves the goal of enhancing our understanding of the nature of and processes involved in numerical cognition.
Finally, modeling efforts like this potentially enhance our efforts to map the functional structure of the brain. We currently lack the capacity to determine in vivo when neuromodulation has changed the underlying configuration of a local neural network, which hinders our ability to attribute function to structure. This approach offers some first steps toward developing reliable methods for detecting changes to the underlying operation of a given local region supports, thereby refining our efforts to describe what the brain is actually doing at any given time.

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References


