A Dynamic Tradeoff Model of Intertemporal Choice

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Abstract

The delay discounting perspective, which assumes an alternative-wise processing of attribute information, has long dominated research on intertemporal choice. Recent studies, however, have suggested that intertemporal choice is based on attribute-wise comparison. This line of research culminated in the tradeoff model (Scholten & Read, 2010; Scholten, Read, & Sanborn, 2014), which can accommodate most established behavioral regularities in intertemporal choice. One drawback of the tradeoff model, however, is that it is static, providing no account of the dynamic process leading to a choice. Here we develop a dynamic tradeoff model that can qualitatively account for empirical findings in intertemporal choice regarding not only choices but also response times. The dynamic model also outperforms the original, static tradeoff model when quantitatively fitting choices from representative data sets, and even outperforms the best-performing dynamic model derived from Decision Field Theory in Dai and Busemeyer (2014) when fitting both choices and response times.

Keywords: intertemporal choice; tradeoff model; dynamic models, random utility, discrimination threshold

Introduction

Many human decisions, mundane or momentous, involve choices between outcomes that materialize at different times in the future, ranging from dieting and exercising plans to education and saving decisions. Research on such *intertemporal choices* has a long history and has revealed various behavioral regularities. For example, it was found that large rewards suffer less proportional discounting than small ones do (the magnitude effect; e.g., Green, Myerson, & McFadden, 1997), and that people's preference between options that have different delays can reverse as time passes (e.g., Green, Fristoe, & Myerson, 1994). Various descriptive models have been developed to account for these empirical phenomena. Among them, the tradeoff model (Scholten & Read, 2010; Scholten, Read, & Sanborn, 2014) currently appears to be one of the most promising models since it provides a unified framework for qualitatively explaining a majority of the empirical findings. Most crucially, it can account for the nonadditivity in delay discounting (e.g., Scholten & Read, 2010; Scholten, Read, & Sanborn, 2014), which eludes any model built on the notion of delay discounting.

One drawback of the tradeoff model, however, is its static nature. As a result, it lacks an account of the dynamic process leading to the explicit intertemporal choices. Nevertheless, any decision is a result of some process that unfolds in time. The characteristics of the process affect the final decision as well as process-related variables such as response time. Therefore, a static model provides only an incomplete description of intertemporal choice, and an account of the underlying dynamics is required for a more comprehensive understanding thereof.

In this paper, we propose a modified tradeoff model of intertemporal choice that has a dynamic structure and can thus account for both choice and response time data. We show that this dynamic tradeoff model can qualitatively accommodate key findings in the literature regarding both choice patterns and relationship between choices and response times. In two model-comparison analyses, we further show that the dynamic model can even outperform promising competing models when fitting empirical data quantitatively.

The Tradeoff Model

To account for intertemporal choice, research has for a long time been mainly conceptualized using the notion of delay discounting, according to which the delay of a reward decreases its present subjective value. One major concern of this approach has been to find the most appropriate form of the discount function, which describes how subjective value decreases with increased delay length. A critical assumption in this endeavor is that each option has a discounted utility or present value independent of other competing options. Importantly, this predicts that intertemporal choice should be transitive: if, among three options X, Y, and Z, one chooses X over Y and Y over Z, then he or she should also choose X over Z.

A series of studies by Read, Scholten, and colleagues, however, demonstrated that the transitivity of intertemporal choices is sometimes violated for a triple of options S, M, and L with increasing money amounts and delay lengths (e.g., Scholten et al., 2014). For example, when facing Option S of receiving \$30 in one week, Option M of receiving \$35 in two weeks, and Option L of receiving \$40 in three weeks, some people might choose Option S over Option M, choose Option M over Option L, but choose Option L over Option S. In each pair, one option has a smaller but sooner reward (*smaller-but-sooner*, or SS, option), and the other has a larger but later reward (*larger-but-later*, or LL, option). The cyclical choice pattern suggests that such people prefer the SS options for adjacent pairs of options (e.g., Option S vs. Option M), but the LL option for the distant pair (Option S vs. Option L). By contrast, others may instead choose the LL options for adjacent pairs but the SS option for the distant pair. The former cyclical pattern can be accounted for by assuming that the amount of discounting associated with a given difference in delay (i.e., the interval between one and three weeks) is smaller when it is treated as a whole than when it is divided into subintervals (e.g., two subintervals of one week), whereas the latter implies the opposite. Together, these patterns suggest nonadditivity in delay discounting.

To accommodate violation of transitivity in intertemporal choice, several alternatives to discounting models were developed, culminating in the tradeoff model. The key difference between the tradeoff model and previous delay discounting models lies in how attributes are assumed to be processed. In contrast to the notion of alternative-wise delay discounting, the tradeoff model assumes that people process attribute information in intertemporal choice by comparing options within each attribute, and that advantages on one attribute (e.g., reward amount) are traded off against the disadvantages on the other attribute (e.g., waiting time). Such an attribute-based approach has been shown to better capture the empirical data quantitatively than the traditional alternative-based approach reflected in the delay discounting paradigm (Dai & Busemeyer, 2014).

According to the tradeoff model, when choosing between an SS option with a money amount of x_S and a delay length of t_S, and an LL option with a money amount of x_L and a delay length of t_L, a decision maker (DM) compares the effective compensation with the effective interval. Let v(x) denote a value function and w(t) denote a time weighting function. The effective compensation is defined as the difference in the value of the two money amounts, that is, $v(x_L) - v(x_S)$, and the effective interval is defined as the difference in the weighted delay lengths, that is, $w(t_L)$ - $w(t_S)$. In addition, the effective interval is assumed to be weighed against the effective compensation by a tradeoff function $Q(w(t_L) - w(t_S))$ to make the decision. The SS option should be preferred when $Q(w(t_L) - w(t_S))$ is larger than $v(x_L) - v(x_S)$, and the LL option should be preferred when $Q(w(t_L) - w(t_S))$ is smaller than $v(x_L) - v(x_S)$.

In the latest version of the tradeoff model (Scholten et al., 2014), the subjective value of a money amount *x* is given by

$$v(x) = \frac{1}{\gamma}\log(1 + \gamma x), \qquad (1)$$

where γ represents diminishing absolute sensitivity to differences in money amount, the time weight of a delay length *t* is given by

$$w(t) = \frac{1}{\tau}\log(1 + \tau t), \qquad (2)$$

where τ represents diminishing absolute sensitivity to differences in delay length, and

$$Q\big(w(t_L) - w(t_S)\big) = \frac{\kappa}{\alpha}\log\left(1 + \alpha\left(\frac{w(t_L) - w(t_S)}{\vartheta}\right)^{\vartheta}\right), \qquad (3)$$

in which κ > 0 represents delay sensitivity, ϑ > 1 represents superadditivity, and α > 0 represents subadditivity. To accommodate probabilistic choice patterns (Dai & Busemeyer, 2014), it is further assumed that the choice probability of the LL option over the SS option is given by a ratio rule, that is,

$$\Pr(\mathrm{LL}|\{\mathrm{SS}, \mathrm{LL}\}) = \left(\frac{v(x_L) - v(x_S)}{Q\big(w(t_L) - w(t_S)\big)}\right)^{1/\epsilon}, \qquad (4)$$

where ε > 0 represents response noise. With these assumptions, the models can accommodate a large number of behavioral regularities in intertemporal choice, such the aforementioned magnitude effect, preference reversal, and nonadditivity in delay discounting.

A Dynamic Version of the Tradeoff Model

One important aspect of intertemporal choice that the tradeoff model cannot explain is the recent finding regarding a relationship between choices and response times in intertemporal choice (Dai & Busemeyer, 2014). Specifically, it was found that pairs of options that give rise to extreme choice proportions tend to be associated with faster response times than pairs with more moderate choice proportions. We refer to this relationship as the *fast-and-extreme effect*. Because the tradeoff model is silent on the temporal dynamics underlying intertemporal choice, it lacks an account of this finding. Here we present a modification of the model to equip it with a dynamic structure while keeping its key assumption of attribute-based processing.

As the original tradeoff model, we assume that a DM performs intertemporal tradeoffs by comparing the effective intervals with the effective compensations. However, unlike the latest implementation of the model (Scholten et al., 2014), in the modified version we assume a more straightforward comparison that goes without the mediation of the tradeoff function. Specifically, we assume that $v(x_L) - v(x_S)$ is directly compared to $w(t_L) - w(t_S)$. To accommodate the probabilistic nature of intertemporal choice, we make two further assumptions. First, both the effective compensation and the effective interval are assumed to be random, denoted as $V(x_L) - V(x_S)$ and $W(t_L) - W(t_S)$, respectively, to reflect the uncertainty in these subjective evaluations. Second, it is assumed that a decision is made when the absolute difference between the two (random) quantities is larger than a positive value; otherwise the DM acquires another sample of the effective compensation and interval without accumulating preferences from previous samples. This process continues until a decision can be made. Note that the first assumption echoes the notion of random utility in economics (e.g., McFadden, 1973), while the second assumption is built on the concept of discrimination threshold in psychophysics (Fechner, 1860).

To derive quantitative predictions, we assume that the effective compensation and effective interval follow independent normal distributions, with respective variance proportional to the mean of the distribution. As a result, the difference between effective compensation and effective interval is also normally distributed with

$$\mu = [v(x_L) - v(x_S)] - [w(t_L) - w(t_S)], \tag{5}$$

and

$$\sigma = \sqrt{c\{[v(x_L) - v(x_S)] + [w(t_L) - w(t_S)]\}}, \tag{6}$$

in which c is a proportional constant to be estimated from the data.[1] Given the assumption of non-accumulative sampling until a sufficiently large difference is obtained, the choice probability of the LL option is given by

$$\Pr(LL|\{SS, LL\}) = \frac{\Phi(\frac{\mu-\delta}{\sigma})}{\Phi(\frac{-\mu-\delta}{\sigma})+\Phi(\frac{\mu-\delta}{\sigma})}, \tag{7}$$

in which Φ represents the cumulative distribution function of a standard normal distribution, and δ denotes the smallest positive difference (i.e., the positive discrimination threshold) required to make a decision.

Because the modified model goes without the tradeoff function and the related ratio choice rule (which are critical for the original model to accommodate the nonadditivity in delay discounting), an alternative mechanism is required in order to retain this capability. To this end, we further assume that the discrimination thresholds for choosing the SS and LL options (i.e., δ_S and δ_L) are different, echoing the general idea of decision bias in the literature of choice models (e.g., Busemeyer & Townsend, 1993).[2] In this case,

$$\Pr(LL|\{SS, LL\}) = \frac{\Phi(\frac{\mu-\delta_L}{\sigma})}{\Phi(\frac{-\mu-\delta_S}{\sigma})+\Phi(\frac{\mu-\delta_L}{\sigma})}. \tag{8}$$

To derive predictions on response time distributions for the modified tradeoff model, we assume that the time it takes to assess a sample follows a Gamma distribution with a scale parameter of θ and a shape parameter of 2. Because empirical distributions of response time tend to be single-peaked (rather than monotonously decreasing), we fix the shape parameter at 2 instead of 1. The total response time is assumed to be the sum of time(s) required for all samples drawn until a decision is made, plus a nondecision time. With these assumptions, we analytically derive joint probability density functions for both choices and response times (see Dai, Pleskac, & Pachur, 2016). Such analytical solutions are usually not available for other dynamic choice models. This ends our description of the modified tradeoff model (hereafter the *dynamic tradeoff model*). See Figure 1 for the dynamic structure of the model.

[1] The model performance results (reported in a later section) were virtually the same or worse when the standard deviation instead of variance of the relative distribution was set to be proportional to the mean and/or the tradeoff parameter κ in the static model was incorporated into *w(t)* as a multiplicative constant to put subjective value and time weight on the same scale.

[2] Mathematical proof on the necessity of this assumption for accommodating the relevant phenomena is available upon request.

Explanatory Power of the Dynamic Tradeoff Model

Because the dynamic tradeoff model inherits the assumption of attribute-based processing, it can accommodate several key findings in intertemporal choice, including the magnitude effect, the common ratio effect, and the common difference effect. According to the magnitude effect, larger amounts appear to be discounted at a lower rate than smaller ones. For example, if a DM is indifferent between receiving \$100 now and receiving \$200 in a year, suggesting an annual discount rate of 50%, then the same person would tend to prefer receiving \$2000 in a year to receiving \$1000 now, suggesting an annual discount rate lower than 50%. From an attribute-based perspective, this change in discount rate can be easily explained by noticing that the effective compensation between \$1000 and \$2000 is much larger than that between \$100 and \$200, whereas the effective intervals for the two choice scenarios are just the same.

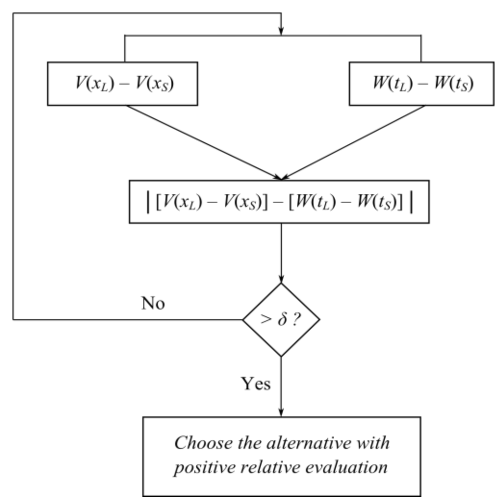

Figure 1: The dynamic structure of the modified tradeoff model of intertemporal choice.

The common ratio effect (i.e., the delay duration effect in Dai and Busemeyer [2014]) implies that increasing the delays of both options proportionally would shift people's preference toward the SS option. In this case, the change in attribute values produces a larger effective interval while keeping the same effective compensation, with the observed effect as a natural result. Finally, the common difference effect suggests that postponing both options by the same length would increase people's preference to the LL option. This effect is accounted for by the attribute-based approach together with the nonlinearity of the time weighting function (i.e., *w(t)*). The particular form of the function (i.e., Equation 2) entails that increasing both delays by the same length would lead to a smaller effective interval and thus shift preference towards the LL option.

With the assumption of distinct discrimination thresholds for choosing the SS and LL options, it can be shown that the dynamic tradeoff model can produce nonadditivity in delay discounting demonstrated as a violation of transitivity.

Specifically, when the probabilistic nature of choice is considered, a violation of transitivity is usually formalized as a violation of weak stochastic transitivity (WST; Davidson & Marschak, 1959). WST requires that for a triple of options X, Y, and Z, if $P_{XY} \geq 0.5$, $P_{YZ} \geq 0.5$, then P_{XZ} should also be no smaller than 0.5, in which P_{AB} represents the probability of choose option A over option B in a binary choice. In other words, a violation of WST occurs when $P_{XZ} < 0.5$ given the two preconditions. This is consistent with the dynamic tradeoff model. For example, for the aforementioned triple of options with increasing money amounts and delay lengths, the dynamic tradeoff model would predict $P_{SM} = 0.53$, $P_{ML} = 0.58$, but $P_{SL} = 0.35$ when $\gamma = 0.05$, $\tau = 0.01$, $c = 1$, $\delta_S = 0.01$, and $\delta_L = 2$, violating WST.

Besides showing intransitive intertemporal choices produced by sub- or superadditivity in delay discounting, Scholten et al. (2014) also suggested a more intricate pattern of intransitive intertemporal choice called relative nonadditivity. Specifically, intransitive intertemporal choices tend to show subadditivity when differences between delay lengths are large relative to the differences between money amounts, and show superadditivity when the differences between delay lengths are large relative to those between money amounts. To account for this pattern, Scholten et al. defined additivity in delay discounting in terms of a product rule of choice odds and showed that this definition naturally led to the pattern of relative nonadditivity. According to this definition, subadditivity occurs when

$$\Omega_{LS} > \Omega_{MS} \times \Omega_{LM}, \quad (9)$$

in which Ω_{XY} denotes the choice odds of option X over option Y, that is, P_{XY}/P_{YX}, and superadditivity occurs when

$$\Omega_{LS} < \Omega_{MS} \times \Omega_{LM}. \quad (10)$$

According to Scholten et al., the ratio choice rule of the static tradeoff model is the key component for explaining relative nonadditivity.

The dynamic tradeoff model, which goes without the ratio choice rule, can account for the same phenomenon. According to the dynamic model, Ω_{MS} and Ω_{LM} tend to be smaller than 1 when differences between delay lengths are large relative to the differences between money amounts, so is Ω_{LS}. Given the choice rule of the dynamic model (i.e., Equation 7 for equal discrimination thresholds for choosing the SS and LL options, or Equation 8 for distinct discrimination threshold), it can be shown that the same conditions tend to render $\Omega_{LS} > \Omega_{MS} \times \Omega_{LM}$. For example, for a triple of options X, Y, and Z with increasing reward amounts of 10, 11, and 12 dollars, and increasing delay lengths of 5, 10, and 15 days, $\Omega_{MS} = 0.074$, $\Omega_{LM} = 0.105$, but $\Omega_{LS} = 0.026 > 0.008 = \Omega_{MS} \times \Omega_{LM}$ when $\gamma = \tau = 0.05$, $c = 1$, and $\delta_S = \delta_L = 0.05$. To the contrary, with the same set of model parameters but another triple of options X', Y', and Z' with increasing reward amounts of 10, 20, and 30 dollars, and the same increasing delay lengths of 5, 10, and 15 days, $\Omega_{MS} = 3.15$, $\Omega_{LM} = 2.30$, but $\Omega_{LS} = 4.14 < 7.26 = \Omega_{MS} \times \Omega_{LM}$. In the first triple, the differences between delay lengths are large relative to those between reward amounts, whereas in the second triple, the latter are large relative to the former.

Besides accounting for major empirical regularities in choice, the dynamic tradeoff model can also accommodate the fast-and-extreme effect, one robust relationship between choices and responses in intertemporal choice (Dai & Busemeyer, 2014). According to the model, the more strongly the expected difference between effective compensation and effective interval differs from zero, the higher the probability of obtaining a difference large enough in each sample and the further away ratio of $\Phi(\frac{\mu-\delta}{\sigma})$ to $\Phi(\frac{-\mu-\delta}{\sigma})$ is from 1. The former leads to faster response times because fewer samples are required to trigger a decision, whereas the latter leads to more extreme choice proportions.

In summary, the dynamic tradeoff model can qualitatively accommodate all the major findings in intertemporal choice that are captured by the static tradeoff model; in addition, it can also qualitatively accommodate the fast-and-extreme effect, a prominent relationship between choices and response times that eludes the static tradeoff model. In the next section, we show further that the dynamic model can also quantitatively fit empirical data better than promising competing models.

Quantitative Model Comparisons

We conducted two model-comparison analyses to show the power of the dynamic tradeoff model in quantitatively fitting empirical data. First, we compared it with the latest, full version of the static tradeoff model (Scholten et al., 2014) in terms of their performance in fitting choice data only. Second, we compared the dynamic tradeoff model with the best-performing model in Dai and Busemeyer (2014)—which is built on Decision Field Theory (DFT; Busemeyer & Townsend, 1993)—with regard to their performance in fitting choice and response time data simultaneously. The DFT model assumes a sequential sampling approach and an attention shift mechanism for making intertemporal choices. Specifically, it suggests that a DM attends to either the money or the delay attribute at a time and evaluates the relevant difference between options to update his or her preference. This preference updating process continues over time as the DM switches attention between the two attributes until the preference level of one option reaches a preference threshold to trigger a decision. See Dai and Busemeyer for more details of the DFT model.

Method

We used data from three representative empirical studies to assess the performance of the models in accounting for individual-level data. The first data set came from Study 1 in Dai (2014), in which half or all the choice questions for each individual had an immediate SS option. The second data set came from Dai (2016), which focused on the nonadditivity in delay discounting and involved only delayed SS and LL options. The third data set came from

Study 3 in Dai and Busemeyer (2014), which examined the magnitude effect, the common ratio effect, and the common difference effect, and again involved only delayed SS and LL options. All three data sets contained participants who showed the fast-and-extreme effect. A total of 138 participants contributed data to the analysis: 61 from the first data set, 40 from the second, and 37 from the third. In all three studies, the choice questions for each participant were adjusted to suit the time preference level of the individual, and each question was presented multiple times. In this way, moderate choice proportions could be induced at an individual level to better distinguish probabilistic models from one another.

The models were fitted to individual data from each data set using the predicted functions of choice probability or joint probability density functions of choices and response times. We used the SIMPLEX algorithm implemented in the fminsearch function of Matlab to find the maximum-likelihood parameter estimates of each model, which was then used to calculate the Bayesian Information Criterion (BIC; Schwarz, 1978). The BIC is a common measure for relative model performance and expresses a model's ability to capture the data, taking into account its complexity (based on the number of free parameters). A lower BIC indicates a better balance between goodness of fit and model complexity and thus a more desirable model.

To evaluate the absolute performance of the dynamic tradeoff model, we compared its predictions with the observed data in terms of the fast-and-extreme effect. Specifically, we categorized all repeatedly presented questions into five equal-interval groups regarding observed choice proportions of the LL options and then calculated the mean observed and predicted response times for each question. The observed and predicted response times within each bin were then averaged to obtain overall measures of the observed and predicted results regarding response time. The fast-and-extreme effect suggests that mean response times associated with moderate choice proportions should be longer than those with extreme choice proportions.

Results

Table 1 presents the results of comparing the static and dynamic tradeoff models in terms of the numbers of participants whose data were better described by either model when fitting only choice data, whereas Table 2 shows the results of comparing the dynamic tradeoff model with the best-performing DFT model in Dai and Busemeyer (2014) when fitting both choice and response time data. In each comparison, the dynamic tradeoff model outperformed the other model both separately for each data set and aggregated across all data sets.[3] Furthermore, Figure 2 shows that the dynamic tradeoff model reproduces the observed fast-and-extreme effect, supporting the validity of the model as a descriptive account. The difference in mean response time between questions with extreme choice proportions (i.e., $p < 0.2$ or $p > 0.8$) and those with moderate choice proportions (i.e., $0.2 \leq p \leq 0.8$) was statistically significant for both observed (t = -9.83, p < .001) and predicted data (t = -5.08, p < .001).

[3] Overall BICs across participants showed the same pattern.

Table 1: Number of Participants Whose Choice Data Were Better Described by the Static or Dynamic Tradeoff Model.

Data Set	Static model	Dynamic model
1	10	51
2	1	39
3	18	19
Across	29	109

Table 2: Number of Participants Whose Choice and Response Time Data Were Better Described by the Best-Performing DFT Model in Dai and Busemeyer (2014) or the Dynamic Tradeoff Model.

Data Set	DFT model	Dynamic tradeoff model
1	18	43
2	14	26
3	15	22
Across	47	91

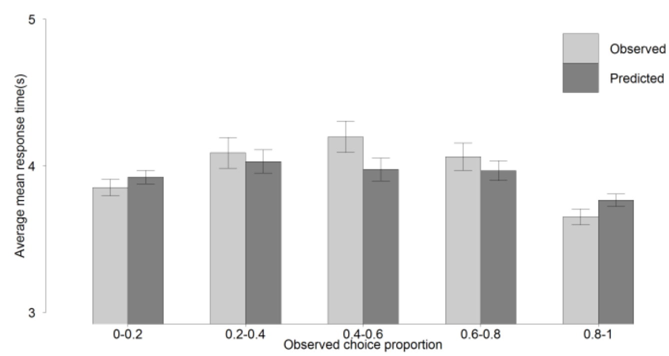

Figure 2. Average mean response times for questions with different observed choice proportions of the LL options. Error bars show 95% confidence intervals.

Discussion

The static tradeoff model (Scholten et al., 2014) represents one of the most successful cognitive models to describe intertemporal choice. However, up to now there have been no attempts to examine how this modeling approach could be extended to also account for the dynamics of the underlying decision process. Here we developed a dynamic modification of the tradeoff model, which can accommodate not only key choice regularities but also the response time data and prominent regularities therein (e.g., the fast-and-extreme effect). We also showed that this modified model quantitatively outperforms the original static tradeoff model when fitting choice data and the best-performing DFT model in Dai and Busemeyer (2014) when fitting both choice and response time data. The model's ability to capture the data both qualitatively and quantitatively underlines the value of developing dynamic accounts of intertemporal choice for a better understanding of this central topic in both psychology and economics.

A General Framework for Developing Dynamic Models of Choice

When developing the dynamic tradeoff model, we invoked and combined two time-honored concepts: the notion of random utility in economics and the concept of discrimination thresholds in psychology. Combining these concepts seems to offer a promising, but so far neglected, approach to developing dynamic choice models, and we argue that it could be applied to transform existing static models of choice also in other domains. For example, it could be applied to extend static models of risky choice into dynamic ones as long as the corresponding models can reasonably offer a measurement of the relative attractiveness of each option and the variability thereof. With dynamic models, both choice and response time data from empirical studies can be utilized to compare competing models for a more powerful model selection. Dai, Pleskac, and Pachur (2016) conduct a more comprehensive development and analysis of such a *random-utility-with-discrimination-threshold* (RUDT) framework, and compare it to other dynamic approaches to modeling intertemporal choice.

Future Directions

In addition to the fast-and-extreme effect discovered in Dai and Busemeyer (2014), recent studies (Dai, 2014) have suggested another striking but less common relationship between choices and response times in intertemporal choice. Specifically, it was found that, within each choice question, the option chosen more frequently also tended to be chosen more quickly than the other option. Unfortunately, this *fast-and-frequent effect* poses a severe challenge to both the best-performing DFT model in Dai and Busemeyer and the dynamic tradeoff model developed here. Both models predict that the conditional response time distribution given choosing one option should be identical to that given choosing the other option. As a consequence, the option chosen more frequently is predicted to have the same mean response time as the other option, contradicting the fast-and-frequent effect. It is possible, however, to modify the dynamic tradeoff model to accommodate this effect (Dai et al., 2016). Specifically, by assuming that the discrimination thresholds are not fixed across successive samples but converging, it is possible to account for the pattern. To put DFT models of intertemporal choice on equal footing, attempts should be made to improve them as well. Future research should explore alternative forms of the tradeoff model under the RUDT structure and compare them with appropriate competing models to examine the performance of the dynamic tradeoff model.

Conclusion

Most existing models of intertemporal choice, including the original tradeoff model, are static and thus lack a proper account of the dynamic processes leading to a choice. In this paper, we showed how the static tradeoff model can be modified into a dynamic one with a general structure built on the concepts of random utility and discrimination threshold. The advantages of the dynamic tradeoff model are demonstrated by its capability to qualitatively accommodate empirical findings and its better performance in quantitative model comparisons. Future studies should further explore the capacity of this approach for explaining more phenomena in intertemporal choice and beyond.

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