

# A Nonlinear Dynamical Systems Theory Perspective on Dual-Processing Accounts of Decision-Making under Uncertainty

Marieke M. J. W. van Rooij (m.vanrooij@bsi.ru.nl)

Behavioral Science Institute,  
Radboud University Nijmegen, the Netherlands

Luis H. Favela (luis.favela@ucf.edu)

Department of Philosophy, Cognitive Sciences Program  
University of Central Florida, USA

## Abstract

Dual-processing accounts of reasoning have gained renewed attention in the past decade, particularly in the fields of social judgment, learning, and decision-making under uncertainty. Although the various accounts differ, the common thread is the distinction between two qualitatively different types of reasoning: explicit/implicit, rational/affective, fast/slow, etc. Consequently, much research has focused on characterizing the two different processes. Less extensive are the attempts to find mediators that influence which process is used. In this paper, we argue that the missing perspective on these dual-processing theories is rooted in dynamical systems theory. By shifting the perspective to the dynamic interaction and transitions between different types of reasoning, we provide a theoretical framework for dual-processing with an emphasis on phase transitions. As a special case, we focus on dual-processing in decision-making and judgment under uncertainty for which we will propose suggestions for future experimental evaluation.

**Keywords:** decision-making under uncertainty; dual-processing; nonlinear dynamical systems theory; phase transitions

## Dual-Process Theories of Reasoning

Dual-processing accounts of reasoning go back to one of the oldest ideas in psychology, namely, that the human mind is not driven by a single, unified process but by two. Exemplars of such ideas include Plato, Hermann von Helmholtz, William James, and Sigmund Freud (Frankish & Evans, 2009).

The history of psychological frameworks for conceptualizing capacities of the mind—especially in terms of decision-making and reasoning—culminated in the 1970s with *dual-process* theories. Dual-process theories have taken on various forms (e.g., see Evans, 2008 for a review). Nevertheless, there are some common features. First, these theories tend to explain the

working of the human mind in terms of two qualitatively distinct cognitive systems, and are referred to as type 1/type 2 (Goodwin & Wason, 1972), System 1/System 2 (Stanovich, 1999), or intuitive/deliberative (Kahneman, 2003). Moreover, these two kinds of cognitive systems tend to be differentiated along the following dichotomies: unconscious/conscious, fast/slow, automatic/controlled, emotional/rational, intuitive/rule-based, etc. There have been, and continues to be, broad applications of these dualistic frameworks. These include, but are not limited to, social judgment (Doherty & Kurtz, 1996), stereotyping (Bodenhausen, Macrae, & Sherman, 1999; Wilson, Lindsey, & Schooler, 2000), learning and problem-solving (Sun, Slusarz, & Terry, 2005), creative thinking (Allen & Thomas, 2011), and perhaps most famously, decision-making and judgment under uncertainty, which we will discuss below as a special case.

The aforementioned phenomena have been investigated and explained in terms of two distinct cognitive systems or processes. However, although they are all “dual-process” theories, many of the models utilize slightly different characterizations of what is being “dual-processed.” For the purposes of the current work, we will utilize “dual-processing” as referring to *System 1*, which is fast-working, implicit, and affect-related, and *System 2*, which is slow-working, explicit, and, analytic.

## Criticism

Dual-processing theories of reasoning are not without opposition (for a recent discussion, see Evans & Stanovich, 2013). Here we highlight three points of criticism we think are most prominent and crosscutting. We refer to these as *conceptual vagueness*, *evidence*, and *transition*.

Perhaps the most fundamental challenge facing dual-process theories is a lack of conceptual clarity. As noted above, although for

current purposes we refer to dual-processing phenomena in terms of System 1 and System 2, there are a number of other labels that are readily used, e.g., *Type 1* and *Type 2*, or *intuitive vs. deliberate processing*. In addition, although the features that fall under each category can be delineated (e.g., Evans & Stanovich, 2013; Frankish & Evans, 2009), it is unclear whether the literature is consistent in the use of such categorizations and if the features are agreed upon. For example, the term *unconscious* is readily used as a feature describing System 1, and *conscious* as a feature describing System 2. Though, what does conscious and unconscious even mean? Theoretical terms that refer to the domain of the “mental” are particularly burdened with the need to be conceptually clear precisely because they are inaccessible directly to third person, objective scientific practice. The lack of such clearly defined and operationalized terms is a particular weakness of dual-process theories that rely on one process being slow and conscious, whereas the other is fast and unconscious.

Another critique of dual-process theories stems from a lack of sufficient empirical evidence. This point is interwoven with the previous critique stemming from conceptual vagueness. It is difficult to have evidence for something if it is unclear what that something is. If we do not know what it is to be conscious or unconscious, it is tough to say that data is evidence of it. Along these lines, without clear conceptual distinctions between System 1 and System 2 features, the question will remain whether there is sufficient empirical evidence to support the strong dichotomy (e.g., Kruglanski & Gigerenzer, 2011; Osman, 2004).

The final critique is the unwarranted strong focus on static rather than dynamic properties of reasoning. Scarcely any attention has been directed to the *transition* from one form of processing to another. The current understanding of factors that influence dual-processing strictly focus on static and binary relationships such that the factors necessary to cause one system to activate do not cause the other system to activate. For example, *meta-cognitive* difficulty has been found to activate System 2 (Alter, Oppenheimer, Epley, & Eyre, 2007), but it is unknown how or exactly when the activation occurs.

Note that it is not our aim to merely reiterate the existing criticisms to dual-processing models. Our starting point is the phenomenon. For example, the difference between calculating two plus two and 17 times 24 is that for most of us,

the latter likely involves a combination of mental gymnastics, finger counting, and putting pen to paper whereas the answer to the first immediately comes to mind (cf. Winerman, 2012). Consequently, instead of discussing questions about the validity of dual-processing as a model for cognitive performance, we propose an entirely different take. We propose treating the transition between different types of cognitive processes in terms of nonlinear dynamic change, which, as we will show, makes the discussion about the strict dichotomy of dual-processes theories obsolete.

### **Our Proposal: A Nonlinear Dynamical Systems Theoretical Approach to Dual-Processing**

Although there are several theoretical and methodological challenges facing dual-process theories of reasoning, there have been strides made towards more nuanced and sophisticated accounts. Evans and Stanovich (2013) put forward a theoretical approach to dual-processing in terms of default responses that can be intervened upon by higher-order reasoning processes. This account provides a potential response to the transition critique discussed above and avoids the pitfalls associated with models that place features of cognitive processing necessarily within a framework of binary categories. Instead, there are *types* of processing that share System 1 or System 2 characteristics. Cognitive processing can occur in both kinds of properties—for example, conscious and unconscious—such that it is not the binary nature of properties that distinguish the kinds of thinking so much as the processes underlying the thinking that differentiates them.

Additionally, Wastell (2014) has attempted to address the issue of conceptual vagueness by appealing to the concepts of *complexity theory*, particularly the notion of *emergence*. Wastell’s is a theory of human reasoning that tries to bring together emergence and modularity theory. He asserts that reasoning, “is a product of our interaction with the environment and our innate ability to create, store, and utilize virtual reasoning modules” that “emerge” from module-environment information fit (2014, p. 354-355).

As Evans and Stanovich prudently note, there is a lot of work still to be done, as the development of dual-process theories—not to mention human reasoning in general—is an evolving project that is likely to continue to develop (2013). In the spirit of contributing to

this evolving project, we present a nonlinear dynamical systems theory perspective on qualitative transitions in human reasoning, with emphasis on the case of decision-making under uncertainty.

### Nonlinear Dynamical Systems Theory

A system can be thought of as a set of parts that have relationships to one another, especially when the integrated elements accomplish a shared or defined task (Norman & Kuras, 2006). Almost all systems are *dynamic*, i.e., their behavior changes over time. Dynamical systems theory refers to the mathematical understanding of change and stability in dynamic systems. Dynamical systems theory provides the tools to describe, model, and evaluate interactions and transitions between qualitatively different behaviors without demanding the definition of qualitatively different systems. Dynamic systems behaviour can be *nonlinear*, i.e., the output is disproportionately related to the input due to multiplicative interactions between its components (Carello & Moreno, 2005; van Rooij, Nash, Rajaraman, & Holden, 2013). This can result in sudden unexpected qualitative transitions from one stable behavior to the next. These kinds of transitions are particularly relevant for a different understanding of dual-processing phenomena in reasoning.

To illustrate how nonlinear dynamical systems theory can shed new light on dual-processing phenomena in reasoning, let us take a look at the transition of ice to liquid water. At certain critical values of pressure and temperature (e.g., 0° Celsius at sea-level), the tiniest change in either can cause the ice to rapidly become liquid whereas at other values a much larger change may not result. This change between two qualitatively different states of water is called a *phase transition*. Phase transitions can be anticipated or accompanied by universal dynamical patterns, so-called *catastrophe flags* (Gilmore, 1993; Isnard & Zeeman, 1976; Thom, 1972). For example, although common understanding is that the transition of liquid back into ice occurs at 0° Celsius, in reality water does not freeze until it is -4° Celsius. This means that between 0° and -4° Celsius, water is *multistable*, and the transition of ice into water—and vice versa—does not necessarily occur at the same temperature value. Moreover, a so-called *hysteresis* effect is observed: After ice turns into water due to increasing temperature, a significantly larger decrease in temperature is needed to re-establish the previous solid state.

Sudden change, multistability, and hysteresis are three of the eight known catastrophe flags (Gilmore, 1981). All eight catastrophe flags are behavioral properties of nonlinear dynamical systems *at the level of their dynamics*, although some (e.g., hysteresis and multistability) can also be observed at the level of the system behavior itself. Observing catastrophe flags around qualitative change is a strong indicator that the system of interest is a nonlinear dynamical system, and that the observed change is indicative of a phase transition. Catastrophe flags have been observed in relation to a broad range of human behavioral changes. The most extensive and successful applications of nonlinear dynamical systems theory to human behavior are to human development (e.g., Thelen & Smith, 1998; van Geert, 1994), movement, action, and perception (e.g., Fajen & Warren, 2003; Haken, Kelso, & Bunz, 1985; Richardson, Marsh, Isenhower, Goodman, & Schmidt, 2007; Warren, 1984).

Applications to human higher-order cognition are less numerous for at least two reasons: First, the dominant computational-representational paradigm in cognitive science resulted in a strong focus on stable states rather than change (cf. Stephen & Van Orden, 2012). Second, in addition to issues related to localizing cognitive functions in the brain (cf. Anderson, 2014), defining precise boundaries around cognitive systems (e.g., Chemero, 2009; Favela & Chemero, 2016) and properly accounting for relevant degrees of freedom in cognitive systems can be very challenging (cf. Kelso, 1995). There have a number of successes though. One particular empirically compelling demonstration of a higher-order cognitive phase transition is by Van Orden and colleagues (1999), who identified homophones as a bifurcation phenomenon.

In the following section, we present a case study demonstrating the application of nonlinear dynamical systems theory with an emphasis on phase transitions to inform a novel approach to dual-processing in decision-making. It is important to note that the concept of a “phase transition” is not merely a metaphor for physical change; it refers to *real* change in a system. A phase transition is nature’s mechanism of qualitative change. It provides a way to explain the existence of qualitatively different behaviors within a continuum.

## Case Study: Dual-Processing as Phase Transition in Decision-Making and Judgment under Uncertainty

As noted above, that humans have multiple decision-making capabilities is an idea that has reoccurred over the history of psychological theorizing. One commonality among such theories (e.g., Plato, Helmholtz, Kahneman, etc.) is that these multiple kinds are distinct, modular, and cognitively encapsulated processes. Such a treatment is faced with serious theoretical and methodological challenges stemming from issues related to conceptual vagueness and lack of empirical evidence. One particular criticism, which we think has not received enough attention, stems from understanding the transitions among these various capabilities. Our main claim is that such transitions are best understood as *phase transitions*.

### Suggestions for Experimental Evaluation

Taking a nonlinear dynamical systems theoretic view on dual-processing phenomena in decision-making under uncertainty means we need to study the transitions in cognitive processing along the continuum of reasoning from ultra-quick memory retrieval to slow explicit processing. This requires finding the transition itself, which includes identifying order and control parameters that characterize the underlying system.

**Finding the Transition.** In a clever series of experiments, Alter et al., (2007) devised ways in which they could activate their version of System 2. For example, they found that participants presented with information that was difficult to read were less prone to reasoning biases typically associated with System 1 and concluded that incidental experiences of difficulty or “disfluency” activated System 2. Unfortunately, in each of the experiments, only two conditions were used. The obvious extension to this type of work would be to present participants with a gradual change in legibility of the presented information in order to find the actual transition, and see whether this is indeed discontinuous (as dual-processing theories suggest) or continuous.

**Finding the Order and Control Parameters.** *Order parameters* capture macroscopic states of a system (Haken, 1988/2006). These order parameters determine the behavior of individual parts of the system. Order parameters can also be

thought of as the dependent variable of a system, that is, the target of measurement in an experiment. *Control parameters* are variables that guide a system’s dynamics. Control parameters can also be thought of as independent variables of a system. Note however that although it can be helpful to think of control parameters as independent variables and order parameters as dependent variables, it is important to keep in mind that the relationship between order and control parameters is not exactly the same as that of dependent and independent variables. The control variable does not *cause* the system’s behavior. This is partially due to a “slaving” relationship between order and control parameters (Haken, 1988/2006). Although control parameters are intertwined with the dynamics exhibited by the order parameter, the relationship is nonlinear.

Identifying the order and control parameters allows for the development of models that capture the full range of a system’s dynamics. In the same way that the interaction between pressure and temperature results in phase transitions of water, we must therefore discover variables that lead the cognitive system through its possible states in decision-making. Following Thelen and Smith (1998), such a task can be summed up as follows: First, identify the order parameter of interest. Second, characterize the qualitatively different behaviours corresponding to different values of the order parameter. Third, describe the dynamic trajectory of the order parameter. Fourth, identify points of transition. Finally, identify potential control parameters and manipulate putative control parameters to experimentally generate transitions.

Recently, we utilized these steps to demonstrate catastrophe flags in the behavioral transition between risk-seeking and risk-averse strategies in a simple risky choice task (van Rooij, Favela, Malone & Richardson, 2013a, 2013b) and similarly in the preference between immediate and delayed rewards (van Rooij & Richardson, 2014). The success of these studies bolsters the case for taking nonlinear dynamical systems theoretic approach to dual-processing.

Another factor hypothesized to affect decision making in terms of dual-processing is *cognitive load*. Cognitive load refers to the demand that cognitive activities place on working memory (e.g., Miller, 1956). When System 1 is under heavy cognitive load, its ability to correct System 2 decreases, which can lead to more unexpected decision behavior. For example, increased cognitive load results in higher than

expected levels of risk aversion (Benjamin, Brown, & Shapiro, 2013). We hypothesize that cognitive load may act as a so-called *splitting factor*, i.e., an order parameter influencing the moment of occurrence of the phase transition.

### Discussion & Future Directions

Attempts to characterize the workings of the human mind in terms of two cognitive systems or processing, as well as debates about the conceptual strength of these characterizations, distract from a more fundamental issue: Starting with the phenomenological distinction between slow/deliberate and fast/intuitive reasoning, it is the interactions between these processes as well as the dynamic control that should be the focus of scientific inquiries into the subject. Accordingly, this requires an approach rooted in nonlinear dynamical systems theory.

Nonlinear dynamical systems are also referred to as *complex systems*, and are formed from (many) parts such that the behavior of the system is irreducible or deducible from the properties of the parts (Norman & Kuras, 2006). In other words, the behavior of complex systems is dominated by the dynamic interactions between parts rather than the properties of the parts themselves (Van Orden, Holden, & Turvey, 2003, 2005).

Most natural systems are complex in the ways just described. A clear example is the human brain. Not only is brain activity constantly interacting and controlled by behavioral and environmental factors (e.g., Van Orden, Hollis, & Wallot, 2012), but emerging, global-level properties such as consciousness are not easily captured in one-to-one relations between single neurons or even single areas of the (Anderson, 2014). A growing body of literature indeed places human cognition firmly within a framework of nonlinear dynamical systems or complex systems theory. We believe dual-processing theories could also be understood within this framework and outlined possibilities for experimental evaluation.

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