Visual bias of diagram in logical reasoning

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Abstract

We analyze the information discrepancy between diagrammatic representations and logical reasoning, which we call *visual biases in diagrammatic reasoning*. Diagrammatic representations contain semantic information, which is based on the topological configurations of objects, and visual information, such as geometric location. In principle, visual information is unnecessary to the validity of logical reasoning. However, people are so sensitive to visual information such as size and shape in diagrams that they occasionally do not ignore irrelevant information. This phenomenon leads to mistakes in logical reasoning. We addressed this issue in the present study. In Experiment 1, we assessed whether and how a visual bias of external diagrams affects reasoning performance. We asked participants to directly manipulate size-fixed (Euler) diagrams while solving syllogistic tasks. In Experiment 2, we tested whether size-scalable diagrams were able to reduce a visual bias of diagrams in logical reasoning.

Keywords: diagrammatic reasoning; external representation; visual bias; diagram particularity; diagram layout; logical reasoning

Introduction

There is a strong belief that diagrammatic representations always facilitate logical reasoning such as syllogisms. However, diagrammatic representations sometimes impede logical reasoning, even when they correctly express the information contained in premisses. This is partly due to the information discrepancy between diagrammatic representations and logical reasoning. We call this *visual biases in diagrammatic reasoning*. Diagrammatic representations contain semantic information that is based on the topological configurations of objects, and visual information, such as geometric location. For example, consider a diagram like the symbol of the Olympic Games. There is a common part between the blue circle and yellow circle, and the blue circle is located to the left of the yellow circle. On the other hand, the validity of a logical inference can be determined only by semantic information of logical connectives (*and*, *or*) and quantifiers (*all*, *some*), and not by words with particular contents (*dog*, *frog*). It is possible to express the logical semantic relationships as topological configurations of diagrammatic objects (cf. Allwein & Barwise, 1996). Consequently, in principle, the visual information contained in a diagram, such as geometrical features, is not necessary to check the validity of a logical argument. However, people are so sensitive to visual information such as size and shape in diagrams (cf. Treisman, 1988) that they occasionally do not ignore irrelevant information. This is a source of errors in logical reasoning.

In a notable study within the cognitive science literature, Stenning and Oberlander (1995) proposed the concept of *specificity* to analyze (in)effectiveness of diagrammatic representations in logical reasoning. In their theory, diagrams can specify certain classes of information in that one diagram corresponds to one model. In contrast, sentential representations do not have this property of specificity since one sentence can represent multiple models. This distinctive property of semantics in diagrammatic representations can affect various processes of diagrammatic reasoning. For example, it enables us to *directly* extract (interpret) semantic information from diagrams (Stenning & Lemon, 2001). On the other hand, diagrams with the property of specificity can be ineffective in handling multiple possibilities by combining diagrams (Shimojima & Katagiri, 2013). In such cases, more attention must be paid to the semantic information than the visual information.

Our view regarding the visual biases of diagrams is consistent with that discussed in a recent Human-Computer Interaction study of diagram layout (cf. Purchase, 2012). In this study, the researchers used semantically equivalent but visually different diagrams. Using such diagrams, the researchers were able to study the way in which the visual layout of a diagram could impede the extraction (interpretation) of semantic information. Indeed, in an early study, Purchase (1997) found that the number of edge crossings in a graph (node-link diagram) had a negative effect on human graph reading. More recently, Benoy and Rodgers (2007) found that jagged shapes were detrimental to Euler diagram (set diagram) comprehension. Additionally, Blake, Stapleton, Rodgers, Cheek, and Howse (2012) explored the role of diagram orientation by comparing participant comprehension of the Euler diagrams at several angles. The findings in the above studies were limited to the process of information extraction from diagrams. In the present study, we aim to explore visual biases of diagrams in *reasoning* or *inference*, using semantically equivalent but visually different diagrams. We focus on visual layout, which can impede the process of information transformation (inference) process from given diagrams.

This approach enables us to go beyond visual biases of diagrams. One traditional technique is described in a seminal study by Barwise and Etchemend (1994). Their "Hyperproof" system is a computer-assisted learning program of logic that uses a hybrid interface of logical formulas and diagrams. In hyperproof, blocks are placed on a chess board. The relationships between the blocks are expressed by propositions (formulas). By observing the block diagrams, the truth of a proposition can be checked and logical relationships between propositions can be validated. In this set up, some blocks (e.g., cylinders) do not have specific properties (shape, size), and thus work as variables. As a result, at least their system can be free from visual biases of diagrams technically. However, it is important to note that diagrams do not indicate whether a visible property (e.g., size) of an object has a meaning in the hyperproof system. In contrast, we propose a way to create diagrams such that they contain objects that work as variables. Specifically, we introduce a method for creating size-scalable diagrams, in which object-sizes are scalable from default, to avoid visual bias of diagrams in logical reasoning.

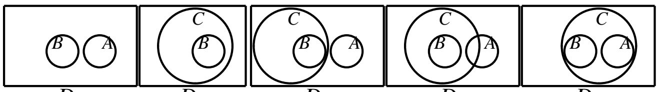
In Experiment 1, we used size-fixed diagrams, to address the question of whether and how a visual bias of diagrams affects reasoning performance. In Experiment 2, we examined whether size-scalable diagrams would be able to reduce visual bias in diagrams representing logical arguments. Before we describe the methods of the two experiments, we will discuss size-fixed and size-scalable forms of Euler diagrams in more detail.

Task analysis

The normal view of Euler diagrammatic representation is that the spatial configuration, such as inclusion and exclusion between circles, represent the set relationships, such as subset and disjoint, which can be translated to categorical sentences, such as *All A are B* and *No A are B* (cf. Sato, Mineshima, & Takemura, 2010a; Mineshima, Sato, Takemura, & Okada, 2014). Here, it is assumed that users deal with the topological configurations of the size-scalable circles rather than their geometric locations. In an empirical study by Sato et al (2010a,b) that investigated the effectiveness of externally provided Euler diagrams in syllogistic reasoning, participants were instructed not to write anything on paper, and were expected to *internally* (not directly) manipulate external diagrams. This was an attempt to focus reasoner attention onto the semantic (topological) information contained in the diagrams. In contrast, we wanted to block the mental conversion of external diagrams and focus reasoner attention not only onto the topological features, but also onto the visual (geometrical) features of Euler diagrams. Thus, we asked the participants to directly manipulate the diagrams (having specific sizes) when solving reasoning tasks. We used the three kinds of Euler diagrams: (1) size-fixed diagrams (type I), (2) size-fixed diagrams (type II), and (3) size-scalable diagrams.

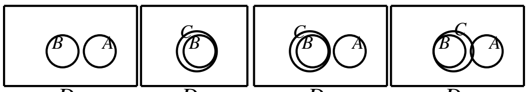
Reasoning with size-fixed diagrams: type I

We start with Euler diagrams that have specific circle-sizes. For example, consider the EA3 syllogism, which has no valid conclusion (NVC). The first premise *No B are A* is represented by D_1 in Fig. 1, and the second premise *All B are C* is represented by D_2. In these two diagrams, circle-sizes are not scalable from the default size of the premise diagrams, such as D_1 and D_2; only circle C is movable. Here the unification of the premise diagrams does not uniquely determine the spatial relationship between circles C and A, i.e., there are three possible configurations of circles C and A (C excludes A; C includes A; C and A partially overlap). A possible strategy for judging NVC is to start to enumerate multiple configurations of conclusion diagrams D_3, D_4, and D_5 until it becomes apparent that there is no relationship between C and A that



D_1 D_2 D_3 D_4 D_5

Fig. 1: An example of reasoning with NVC using size-fixed diagrams: type I



D_6 D_7 D_8 D_9

Fig. 2: An example of syllogistic reasoning with NVC using size-fixed diagrams: type II

holds in all the conclusion diagrams. An alternative strategy is to place the circles C and A such that they partially overlap; i.e., D_4. When such a configuration of circles holds, both *No C are A* and *All C are A* are invalid conclusions. Additionally, when (1) syllogisms consist of universally quantified sentences or (2) the Euler diagrams do not hold the existential assumption for minimal regions, one can determine whether a syllogism is NVC using the partial-overlapping strategy.

When using size-fixed diagrams (type I), all three possible configurations of circles C and A can be exhaustive, and thereby both the enumeration and partial-overlapping strategies are available. Such diagrams could work in a positive way when solving reasoning tasks.

Reasoning with size-fixed diagrams: type II

Consider the other type of Euler diagrams, in which the circles have different sizes. In Fig. 2, the circle-size of C in D_7 (corresponding to the second premise *All B are C*) is smaller than that of C in D_2 of Fig.1. The unification process almost automatically determines the exclusion configuration between circles C and A, as shown in D_8. In addition, the unification process gives rise only to the partial-overlapping configuration as in D_9 as far as the circle C is carefully placed on the most right side. Here, the reasoner has the option to construct the overlap configuration, but this is a difficult task because the circle-size of C is so small. Given this small circle-size, the reasoner can not construct the circle configuration of inclusion relation in conclusion diagram, corresponding to D_5 in Fig. 1.

In this case using size-fixed diagrams (type II), all three possible configuration of circles C and A are not exhaustive. This makes it difficult for reasoner to choose an enumeration strategy. On the other hand, the circles C and A can be placed in such a way that they partially overlap. Thus, one can determine whether a logical argument has NVC using the partial-overlapping strategy with the premise diagrams. However, it is relatively difficult to use this partial-overlapping strategy. This is due to a visual bias in diagrammatic reasoning: i.e., the possible area of intersection between C and A is very small because the circle C (or A) is the smallest size available when constructing the overlap configuration in the conclusion diagram. In such cases, these diagrams might work in a negative way when solving reasoning tasks.

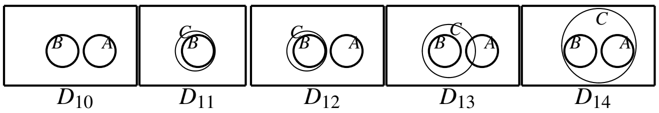


Fig. 3: An example of syllogistic reasoning with NVC using size-scalable diagrams

Table 1: Syllogisms (first and second premises) used in the experiment

Syllogisms having NVC		Syllogisms having VC	
AE1	*All B are A, No C are B.*	AE2	*All A are B, No C are B.*
AE3	*All B are A, No B are C.*	AE4	*All A are B, No B are C.*
EA3	*No B are A, All B are C.*	EA1	*No B are A, All C are B.*
EA4	*No A are B, All B are C.*	EA2	*No A are B, All C are B.*
		AA1	*All B are A, All C are B.*

Reasoning with size-scalable diagrams

Consider the size-scalable Euler diagrams used for solving the EA3 syllogism that has NVC, as shown in Fig. 3. Each default circle-size in the premise diagrams D_{10} and D_{11} is the same as that in D_6 and D_7 from the size-fixed diagrams (type II) in Fig. 2. In the size-scalable diagrams, the reasoner can scale the size of circle C up and down (denoted by thin lines). For the convention, the unification of premise diagrams can give rise to all three possible configurations of circles C and A: an exclusion relationship in D_{12}, a partial-overlapping relationship in D_{13}, and an inclusion relationship in D_{14}.

In this case using size-scalable diagrams, all three possible configurations of circles C and A can be exhaustive. As a result, both the enumeration and partial-overlapping strategies are available. Thus, such diagrams would resist the visual bias found in size-fixed diagrams (type II) as stated in Fig. 2.

Syllogisms having universally quantified sentences

For simplicity, the premises and conclusions of the syllogisms used in this experiment were universally quantified sentences of the form either *All A are B* or *No A are B*. All the syllogisms used in this experiment are shown in Table 1. We gave nine different types of syllogisms in total, out of which 4 syllogisms had no-valid conclusion (NVC) and 5 syllogisms had a valid conclusion (as filler tasks). The NVC types that cannot fix the circle-sizes uniquely (AA2, AA3, EE1, EE2, EE3, EE4) were not used. The valid conclusion of AE2, AE4, EA1, and EA2 syllogisms is *No C are A* (VC-*no*), and that of AA1 is *All C are A* (VC-*all*).

Experiment 1

In Experiment 1, we examined whether and how human diagrammatic reasoning is affected by visual bias of diagrams. We asked participants to directly manipulate size-fixed diagrams (type I for Experiment 1a; type II for Experiment 1b) when solving syllogistic tasks with universally quantified sentences.

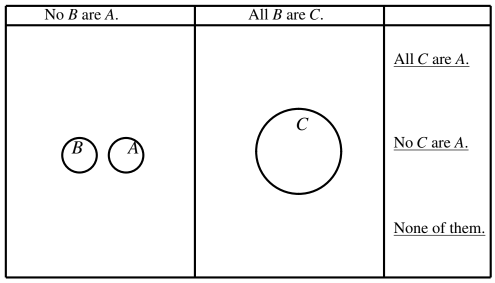


Fig. 4: A syllogistic task with size-fixed diagrams (type I) for Experiment 1a; the correct answer is *None of them*.

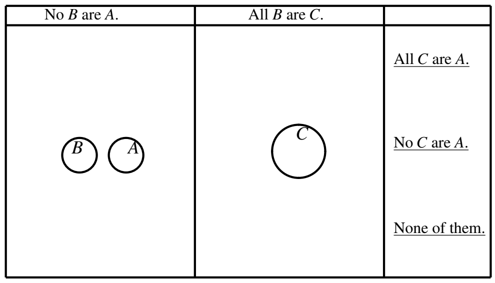


Fig. 5: A syllogistic task with size-fixed diagrams (type II) for Experiment 1b; the correct answer is *None of them*.

Method

Participants Forty-five undergraduate and graduate students from the University of Tokyo (mean age 20.11 $\pm$ 1.32 SD) were recruited by means of a poster placed on the campus. All participants gave informed consent and were paid for their participation. All procedures in this experiment were approved by the Ethics Committee of the University of Tokyo. The participants were Japanese-speaking students, and the sentences and instructions were provided in Japanese. The participants were divided into two groups: Experiment 1a (24 participants) and Experiment 1b (21 participants). In Experiment 1a groups, two participants were excluded–one for a computer malfunction, and one for response times more than two SD from the mean. In Experiment 1b groups, we excluded data from one participant who did not use (move) diagrams.

Materials The participants in the both groups were given syllogisms with (size-fixed) Euler diagrams in a PC monitor. The participants were presented with two premises and were asked to choose a sentence corresponding to the correct conclusion, from a list of three possibilities. The list consisted of *All C are A*, *No C are A*, and *None of them*. When solving such problems, the participants were also asked to use the diagrams appeared at the center of the monitor (the lower side of the premise sentences), and to move circle C to the left side diagram by drag operation using a mouse (the left side diagram cannot be moved). In Experiment 1a, the size-fixed diagrams (type I), shown in Fig. 4, were provided. In Experiment 1b, the size-fixed diagrams (type II), shown in Fig. 5, were provided. Each syllogistic task of VC-*all* and VC-*no* in Experiment 1a was also used in Experiment 1b.

Table 2: The appearance frequencies of circle configurations in syllogisms having NVC

	Exp-1a	Exp-1b	Exp-2
1. inclusion	4.5	0.0	0.0
2. exclusion	6.8	32.5	13.6
3. overlap	**35.2**	**28.8**	**28.4**
4. inclusion & exclusion	**15.9**	**0.0**	**3.4**
5. inclusion & overlap	**11.4**	**0.0**	**2.3**
6. exclusion & overlap	**15.9**	**38.8**	**44.3**
7. inclusion & exclusion & overlap	**10.2**	**0.0**	**8.0**
The sum of 3, 4, 5, 6, and 7	**88.6**	**67.5**	**86.4**
The sum of 4 and 7	**26.1**	**0.0**	**11.4**
The sum of 5 and 6	**27.3**	**38.8**	**46.6**

In Experiment 1a, especially, we recorded how the participants move the Euler diagrams in syllogistic reasoning. For analysis, we defined a configuration that remained for more than 0.3 seconds as a unit of circle-configuration in the conclusion diagram. The following cases were not considered to be units of circle configuration: (1) the default configuration, and (2) unintentional configurations produced by the mouse operation which was too fast to correctly drag the pictures.

The task was firstly displayed such that the premise sentences and diagrams were surrounded by frames, as shown in Figs. 4 and 5. These frames were removed after five seconds, and at the same time the participants were instructed to start changing the position of circle C and solve the syllogistic reasoning tasks. There was no time limit to solve the reasoning tasks.

Procedure The experiment was conducted individually. Firstly, the participants were provided with a one-page instruction on the meaning of Euler diagrams, elaborating on diagrams' syntax–semantics correspondence rather than diagrammatic picture–semantics correspondence. A pretest whether they understood the instruction correctly was conducted; it consisted of the problems on the correspondence between universally quantified (affirmative and negative) sentences and Euler diagrams. Before the reasoning tasks, the participants were given oral instruction on the reasoning task and familiarlized with the mouse operations required for circle manipulation. Note that the experimenter did not provide any specific instruction about how to manipulate circle C. One task example (*All A are B, All B are C; therefore All A are C*) was displayed on a PC monitor. A total of nine different types of reasoning tasks were presented in random order (one of three patterns).

Result and discussion

In the pretest of Euler diagrams, all of the participants chose the correct answers for each item. The appearance frequencies of (combinations of) configurations of circles C and A in diagram manipulation for solving NVC tasks are showed in Table 2. The bold numbers refer to the circle-configurations which are consistent with our description in task analysis. Regarding the NVC tasks in Experiment 1a, 88.6% of the participants chose the expected strategies: 35.2% of the participants chose the partial-overlapping strategy, 26.1% of the participants chose the enumeration strategy (4 & 7), and 27.3% of the participants chose a confounding strategy of the two strategies (5 & 6). Furthermore, in VC-*all* tasks, 100.0% of the participants constructed the circle configuration that C is included in A. In VC-*no* tasks, 96.6% of the participants constructed the circle configuration that C is excluded from A. As we described in the subsection of task analysis, the participants chose (i) the enumeration strategy with multiple configurations of conclusion diagrams and (ii) the partial-overlapping strategy of placing two circles (see also Sato, Wajima, & Ueda, 2014). This suggests that all three possible configuration of circles C and A are exhaustive by the size-fixed diagrams used in Experiment 1a.

Table 3: The accuracy rates for NVC tasks with diagrams in Experiments 1a, 1b, and 2

Experiments	Accuracy rates
1a: size-fixed diagrams (type I)	92.0%
1b: size-fixed diagrams (type II)	76.3%
2: size-scalable diagrams (starting from 1b)	92.0%

As shown in Table 3, the accuracy rate of NVC tasks for Experiment 1b (76.3%) was significantly lower than that for Experiment 1a (92.0%) (Mann-Whitney $U = 143$, $P = 0.023$). The accuracy rates of the other two types (VC-*all* and VC-*no*) were also substantially high. Experiment 1a showed 100% for VC-*all*, and 96.6% for VC-*no*; Experiment 1b showed 100% for VC-*all*, and 100% for VC-*no*. The significant difference between accuracy rates for NVC tasks in Experiment 1a and those in Experiment 1b provides evidence for the claim that size-fixed diagrams (type II) suffer from the visual bias of diagrams in logical reasoning.

All of the incorrect answers of NVC tasks in Experiment 1b were *No C are A*. Accordingly, in Experiment 1b, the circle-configuration of exclusion was constructed in 89.5% of the items (incorrectly) selecting *No C are A* in NVC tasks. As shown in Table 2, additionally, the rate of the participants who constructed the exclusion configuration was 6.8% in the Experiment 1a, but the rate became as high as 32.5% in Experiment 1b ($U = 116$, $P = 0.004$).

Experiment 2

If visual bias is actually present in diagrammatic representations, then we expected that performance on the task in Experiment 1b would be improved when participants used size-scalable diagrams instead of size-fixed diagrams (type II). We designed Experiment 2 to confirm this.

Method

Participants Twenty-five undergraduate and graduate students (mean age 19.32 ± 1.35 SD) were recruited. Three participants were excluded–two for a computer malfunction, and one for response times more than two SD from the mean.

Materials The participants were given syllogisms with (size-scalable) Euler diagrams, shown in Fig. 6. The par-

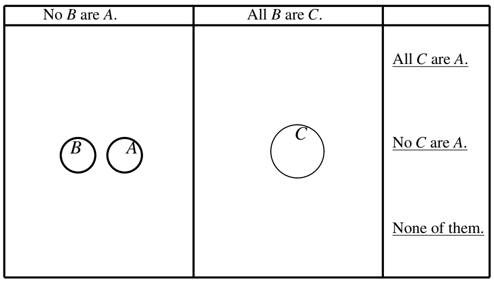


Fig. 6: A syllogistic task with size-scalable diagrams; the correct answer is *None of them*

ticipants were presented with two premises and were asked to choose a sentence corresponding to the correct conclusion. When solving syllogistic tasks, the participants were asked to use the diagrams: move circle *C* to the left side diagram by drag operation using a mouse. In addition, the participants were provided the instruction that the size of circle *C* by scrolling the mouse wheel can be scaled up or down if it is necessary. Each default size of *C* (illustrated by thin lines) is the same as that of size-fixed diagrams (type II) in Experiment 1b. Each syllogistic task of VC-*all* and VC-*no* in Experiment 1 was also used in Experiment 2.

Procedure The experiment was conducted in the same manner as in Experiment 1. The instructions of diagrams were provided, pretests were conducted, and then the reasoning task with diagrams were imposed.

Result and discussion

In the pretest of Euler diagrams, all of the participants chose the correct answers for each item. In the reasoning tasks, nineteen participants scaled up and down the size of circle *C* (three participants did not scale up and down the circle-size in more than two items in NVC tasks). For example, the sixteen participants scaled up the circle *C* in the task of Fig. 6.

The accuracy rate of NVC tasks for Experiment 2 was 92.0%, as shown in Table 3. The data were compared with those of Experiment 1b. The accuracy rate of NVC tasks for Experiment 2 (92.0%) was significantly higher than those for Experiment 1b (76.3%) (Mann-Whitney $U = 140.5$, $P = 0.019$). The accuracy rates of the other two types (VC-*all* and VC-*no*) were substantially high, showing 100% for VC-*all*, and 98.9% for VC-*no*. The significant difference between accuracy rate for NVC tasks in Experiment 2 and that in Experiment 1b provides evidence for the claim that size-scalable diagrams reduce the visual bias in logical reasoning with size-fixed diagrams (type II).

The difference in accuracy rates reflected in the appearance frequencies of circle configurations in Table 2. The number of participants who constructed the exclusion configuration was 13.6% in the Experiment 2, which was lower than 32.5% in Experiment 1b ($U = 144$, $P = 0.039$). Furthermore, in NVC tasks, 86.4% of the participants chose the expected strategies: 28.4% of the participants chose the partial-overlapping strategy, 11.4% of the participants chose the enumeration strategy (4 & 7), and 46.6% of the participants chose the confounding strategy (5 & 6). This suggests that all three possible configuration are exhaustive by the size-scalable diagrams used in Experiment 2.

General discussion

In Experiment 1, we addressed the question of whether and how a visual bias of diagrams affects human performance. We asked participants to directly manipulate size-fixed diagrams when solving syllogistic tasks. In the size-fixed diagrams (type I) used in Experiment 1a, the circle-sizes were sufficient to allow participants to construct configurations of overlap and inclusion in the conclusion diagram. In the size-fixed diagrams (type II) used in Experiment 1b, the circle *C* (or *A*) was set to be the smallest size possible, making it more difficult for participants to construct the configuration of overlap in the conclusion diagram. The difference in accuracy rate for NVC tasks between Experiment 1a and Experiment 1b provided evidence for the existence of a visual bias in diagrammatic reasoning. In Experiment 2, we addressed the question of whether size-scalable diagrams were able to resist visual bias of diagrams. The difference in the accuracy rate for NVC tasks between Experiment 2 and Experiment 1b provided evidence for a reduction of visual bias in diagrammatic reasoning.

In this paper, we focused on visual biases in Euler diagrams representing syllogistic arguments. However, such visual biases would occur in various kind of diagrammatic reasoning, specifically in diagrams that is a linear variant of Euler diagrams (cf. Sato & Mineshima, 2012). In *deductive* reasoning, the size of each individual object in a diagrams can be regarded as a piece of visual information, but these do not necessarily contribute to the validity judgement of deductive reasoning. However, what features of diagrams make the difference between semantic and visual information depends on the type of reasoning. For example, consider *inductive* reasoning. The validity of an inductive argument can be determined by the probability of an event occurrence concerning the truth of the statements. It is possible to express uncertainty relationships through the proportional relationships between diagrammatic objects. This technique is called “area proportional Euler diagrams” (cf. for a survey of various forms of Euler diagrams, see Rodgers, 2014). In this technique, (in contrast to that used in our study) the circle-sizes play an important role in judging the validity of induction (probability judgment). Thus, certain other features of diagrams (e.g., shape, orientation) can be regarded as irrelevant (visual) information.

Concerning the distinction between visual and semantical information, a similar point has been made by the psycholinguistic studies of geometrical richness contained in languages. Most notably, Landau and Jackendoff (1993) analyzed the spatial language such as *The cat is on the mat*, and examined that certain nouns (e.g., *cat*, *mat*) contain much detailed geometrical properties such as shape, comparing with certain prepositions (e.g., *on*). More relevant to our concern is the study of spatial reasoning in Knauff and Johnson-Laird (2002). In their experiments, the response time of inference tasks with visual relations (that are easy to invoke visual representations but hard to invoke spatial representations, e.g., cleaner-dirtier) was longer than that with visuospatial relations (that are easy to invoke visual and spatial representations, e.g., above-below). These results suggest that irrelevant visual detail can impede inferential processes. The above studies dealt with mental images elicited from sentential expression about what we saw. By contrast, this study applies the geometrical richness in language components to that in external diagrams, and further to visual biases of diagrams for solving reasoning tasks.

The issue on visual biases of diagram in logical reasoning is an instance of *diagram particularity* (*generality*) problem, which is well known in the philosophy of mathematics (Kulpa, 2009; Mumma, 2010; Shin, 2012). Indeed, Gottfried Leibniz (1677) wrote:

> we must recognize that these figures [the figures of geometry] must also be regarded as characters, for the circle described on paper is not a true circle and need not be; it is enough that we take it for a circle (p.281).

The diagram written on paper (i.e., external diagram) is just a *particular* object. In principle, a claim constructed from the particular diagram can hold only in the particular case. Thus, there is no guarantee of the correctness on claim in other cases. In other words, they lack *generality*. Therefore, we can not obtain general claim from the particular diagram. This problem influenced early formal studies of diagrammatic reasoning, where the formal definition of a diagram is abstract in that it is independent of any particular diagram (e.g., Shin, 1994). However, if formal studies can contribute the mathematical underpinnings of actual diagrammatic reasoning, then it is important to associate particular diagrams and general diagrams (Howse, Molina, Shin, & Taylor, 2002). This view has especially manifested in the modern development of logic diagrams, such as spider diagrams (e.g., Howse, Stapleton, & Taylor, 2005) and the modern formulation of Euclid's geometry (e.g., Avigad, Dean, & Mumma, 2009). In these systems, primitive objects constituting a diagram such as circles and rectangles are abstractly defined as formal units with *variables*, and the particular diagrams are regarded just as *instances* of diagrammatic objects that comply with the abstract definition. In our empirical study, we suggested size-fixed diagrams as one form of particular diagrams. Our result indicates that human suffer from the problem of visual bias of diagrams (diagram particularity). This is the case not only in special situations, such as mathematical generalization, but also in the everyday use of diagrams. Furthermore, we introduced size-scalable diagrams as objects that play the role of variables; thereby enabling diagrams with specific-size to be recognized as instances of diagrammatic objects. We believe that there is potential for designing pedagogical tools that reduce visual biases in diagrammatic reasoning and learning.

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