The Equiprobability Bias in the Monty Hall Dilemma: A Comparison of Primary School, Secondary School, and University Students

Lore Saenen (Lore.Saenen@ppw.kuleuven.be)
Methodology of Educational Sciences, KU Leuven, Andreas Vesaliusstraat 2
3000 Leuven, Belgium

Mieke Heyvaert (Mieke.Heyvaert@ppw.kuleuven.be)
Methodology of Educational Sciences, KU Leuven, Andreas Vesaliusstraat 2
3000 Leuven, Belgium

Ilke Grosemans (Ilke.Grosemans@student.kuleuven.be)
Methodology of Educational Sciences, KU Leuven, Andreas Vesaliusstraat 2
3000 Leuven, Belgium

Wim Van Dooren (Wim.VanDooren@ppw.kuleuven.be)
Instructional Psychology and Technology, KU Leuven, Dekenstraat 2
3000 Leuven, Belgium

Patrick Onghena (Patrick.Onghena@ppw.kuleuven.be)
Methodology of Educational Sciences, KU Leuven, Andreas Vesaliusstraat 2
3000 Leuven, Belgium

Abstract
The Monty Hall dilemma (MHD) is a notorious brain teaser that received a lot of attention because both novices and statistical experts fail to reason correctly when solving this problem. In the current paper, we try to shed more light on the previous MHD research findings by discussing them in relationship to the equiprobability bias, which is known to develop with age and statistical education. Besides investigating behavioral performances on the MHD, the experiment described in this paper focuses on the level of understanding of the problem and how the latter can be improved. The results show that by increasing the number of alternatives in the MHD, both behavioral performance and understanding of the problem improved. However, full understanding of the MHD was only reached by some participants, and depended on participants’ age and the number of choice alternatives in the MHD.

Keywords: Monty Hall dilemma; equiprobability bias; probability; heuristic reasoning; age differences.

Introduction
The Monty Hall dilemma (MHD) is known as one of the most counterintuitive conditional probability problems. The problem is useful for research in the domain of cognitive psychology because both novices and experts massively solve this problem incorrectly and fail to understand why their reasoning is wrong.

The problem is adapted from an American television show called ‘Let’s make a deal’ (Friedman, 1998) and is named after the host of the program. The classic version of the MHD goes as follows: The host, Monty Hall, gives his contestant the option to choose between three identical doors. A prize is randomly placed behind one of the doors. The other two doors conceal mock prizes, for example goats. After the contestant picks an initial door, the host, being aware of the location of the prize, will not yet open this door to reveal what is behind it. Instead, he will first open another door than the door initially chosen by the contestant to show that it contains a mock prize. At this point, when two doors are left unopened, the host asks the contestant to make a final decision. Thus, the dilemma for the contestant is whether to stay with his initial choice for a door, or to swap to the other remaining unopened door.

Previous research on the MHD has demonstrated that most people have the idea that winning chances are equal for staying and switching (e.g., Franco-Watkins, Derks, & Dougherty, 2003; Granberg & Brown, 1995). Next, there exists a very strong (cross-cultural) tendency to stay with the initial choice (e.g., Friedman, 1998; Granberg, 1999; Granberg & Brown, 1995). When people solve the MHD for the first time, switching rates vary from 4.5% to 21% (e.g., Bown, Read, & Summers, 2003; Friedman, 1998; Page, 1998). Although the majority of the participants believe that switching and staying hold equal winning chances, they feel more inclined to stay with the initial choice because of the larger amount of regret participants anticipate to experience after a loss due to switching compared to a loss due to staying (Stibel, Dror, & Ben-Zeev, 2009).

When applying Bayes’ theorem, however, it becomes clear that the theoretical posterior winning probabilities for staying and switching are unequal. In order to maximize winning chances in the MHD, the best way to proceed is to switch doors because switching yields a 2/3 probability to
win the prize, whereas staying only yields a 1/3 winning probability. Given the strong tendency to stay (cf. supra), this optimal solution is highly counterintuitive.

So far, research has shown that the strong tendency to stay with the initial choice is malleable. For example, when the number of choice alternatives was increased in the MHD, participants showed higher switching rates (e.g., Franco-Watts et al., 2003; Stibel et al., 2009). Next, when participants completed successive trials of the MHD, switching rates increased substantially across trials (e.g., Franco-Watts et al., 2003; Slonembecker & Tyran, 2004).

Despite this improved behavioral performance, however, participants never arrived at optimal behavior, which is to switch on all trials. Furthermore, repeated experience with the problem and the results of the decisions taken improve behavioral performance but do not seem to enhance insight into the underlying probabilities of the MHD; which is referred to as the dissociation between behavioral performance on the MHD and a full understanding of the problem (Franco-Watts et al., 2003; Stibel et al., 2009; Tubau & Alonso, 2003).

One explanation for the lack of understanding of the MHD lies in the equiprobability bias (Lecoutre, 1992), which describes people’s tendency to judge the probability of random events as being equal because “it reflects a process by chance.” In the MHD, this means that the probability of winning is determined by dividing the number of prizes (i.e., one) by the number of remaining alternatives (i.e., two), which leads to the erroneous judgment that staying and switching have posterior winning probabilities that are equally large (e.g., Franco-Watts et al., 2003; Granberg & Brown, 1995). The equiprobability bias then is due to the fact that people heuristically determine the chance of an event by merely considering the number of possible cases (Falk, 1992; Shimo & Ichikawa, 1989).

Previous research showed that the equiprobability bias increases with age (De Neys, 2007) and formal statistical education (Morsanyi, Primi, Chiesi, & Handley, 2009). What would these findings mean for the identified dissociation between behavioral performance on the MHD and the level of understanding of the problem, knowing that the majority of MHD research has been conducted with adult participants? So far, only one study compared MHD performance between participants of different age groups (De Neys, 2007). In his research, De Neys (2007) confronted 13- till 18-year-old participants with the MHD and questioned which behavior (i.e., staying, switching, or chances are equal) would maximize winning chances. First, the results showed that the oldest participants most often chose the ‘chances are equal’ answer, which indicates that they were most influenced by the equiprobability bias. Second, the youngest participants more often gave the ‘switching’ response compared to older participants, and when they not switched, they chose the ‘staying’ response more often than the ‘chances are equal’ answer. The ‘staying’ response is not based on a consideration of the number of possibilities, but has an affective basis: Staying with the initial choice is an anticipation of regret (Gilovich, Medvec, & Chen, 1995; Stibel et al., 2009). The problem with the study of De Neys (2007), however, is that he only included behavior as dependent variable. Consequently, it does not reveal whether the relation between behavioral performance and the level of MHD understanding is the same for younger participants as for adult participants. The present study tries to shed a first light on this research gap.

In the current experiment, participants completed ten trials of an MHD variant. We minimized the influence of regret (cf. infra), because of our primary interest in people’s cognitive reasoning processes on the MHD. First, we investigated whether the MHD was solved differently by participants of different age groups. Second, we manipulated the number of alternatives in the MHD. In the classic version of the MHD, only three alternatives are included. With an increasing number of alternatives, the intuition that the initial choice is likely to be the correct one will be less available, because the difference between the prior probabilities to initially pick the correct versus wrong door becomes more salient (i.e., 1/10 vs. 9/10 in a 10-door MHD variant, compared to 1/3 vs. 2/3 in the classic 3-door MHD). Thereby, one would be less stimulated to stay with the initial choice, and the equiprobability bias would no longer be influential. Next, previous research has shown that in the classic MHD situation, the difference in the posterior winning probabilities when staying or switching (i.e., 1/3 vs. 2/3) is not sufficiently salient for participants (e.g., Franco-Watts et al., 2003; Stibel et al., 2009). With an increasing number of alternatives, this difference will become more salient (i.e., 1/10 vs. 9/10 in a 10-door MHD variant). Furthermore, repeated experience with the MHD will reveal (an approximation of) the underlying switching reinforcement rate, which of course is higher with an increased number of alternatives. As a consequence, participants assigned to a condition with an MHD variant with a higher number of alternatives will experience more ‘winning when switching’ trials and may therefore more easily pick up the advantage of switching.

For the behavioral MHD performances, our predictions were the following: (1) For each age group, increased number of alternatives would result in increased switching rates; (2) For the classic MHD, younger participants would show higher switching rates compared to older participants because the equiprobability bias is less developed.

For MHD understanding, we predicted the following: (1) Participants of the youngest age group would show least understanding of the problem, because understanding the MHD requires some minimal understanding of probabilistic situations (Fischbein & Schnarch, 1997); (2) For older participants, whose understanding of probabilistic situations is at least minimally developed, the highest level of MHD understanding will be observed in the MHD variant with the highest number of alternatives.
Methods

Participants and Design

Three-hundred eighty-five students from three different age groups (primary school, secondary school, and university) participated in the experiment. Ninety-six of them were familiar with the MHD and were excluded from the analyses. Descriptive statistics of the 289 included participants in the three age groups can be found in Table 1.

Participants of each age group were assigned to one of three treatment conditions: An MHD variant with 3 vs. 10 vs. 50 alternatives. As a consequence, this study involves a 3 (age group) x 3 (number of alternatives) between subjects design.

The study protocol was approved by the Ethical Committee of KU Leuven.

<table>
<thead>
<tr>
<th>Age group</th>
<th>Primary school</th>
<th>Secondary school</th>
<th>University</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>100</td>
<td>98</td>
<td>91</td>
</tr>
<tr>
<td>(M_{age})</td>
<td>10.27</td>
<td>14.71</td>
<td>18.85</td>
</tr>
<tr>
<td>(SD_{age})</td>
<td>0.81</td>
<td>0.70</td>
<td>3.94</td>
</tr>
</tbody>
</table>

Materials and Procedure

Five participants of the same age group came to the laboratory simultaneously. One reason to conduct the experiment in groups of five participants was efficiency. Another reason was the diminished influence of regret, because participants would be responsible for the initial choice in only 20% of the MHD trials (cf. infra).

Upon arriving, each participant completed some demographical questions. Next, each participant individually received a sheet of paper to complete during the experimental MHD trials.

The MHD was operationalized by 3, 10, or 50 numbered cups (cf. doors) and one jolly toy (cf. prize) randomly placed beneath one of the cups. Each group of five students was confronted with 11 trials of the MHD, containing one practice trial and ten experimental trials. Before the start of the experiment, the experimenter placed the material for each trial (cups with one jolly toy randomly placed beneath one of the cups) on a separate table (i.e., 11 tables in total).

At the start of the experiment, the experimenter asked the five students to stand in a fixed row, leaving enough space between each other in order to avoid interaction and collaboration between the participants. For the practice trial, the student standing first in the row (i.e., participant 1) was asked to say aloud the number of the cup of which (s)he thought it would contain the jolly toy. All participants were asked to write down this number in the first column of their sheet of paper. Next, the experimenter took away 1, 8, or 48 other cups that did not contain the prize (depending on the treatment condition). Next, the experimenter asked each participant to decide individually whether to stay with the initial choice (made by participant 1), or to switch to the other remaining cup. Each participant was asked to write down his/her choice in the second column by encircling either the word ‘staying’ or ‘switching’. Note that participant 1, who made and communicated aloud his/her initial choice for a cup, was not allowed to say aloud whether (s)he stayed with the initial choice or switched. The experimenter then removed the two remaining cups and communicated the outcome of the trial (e.g., “In this trial, staying with the initial choice for cup number 3 resulted in winning, whereas switching to cup number 1 resulted in losing”). Finally, each participant was asked to write down the outcome of the current trial depending on the choice (s)he made individually to either stay or switch, by encircling either the word ‘winning’ or ‘losing’ in the third column. For the first experimental trial, the same participant as for the practice trial was asked to make the initial choice for a cup. For the subsequent experimental trials, the participant next in the row got to make the initial choice. Thus, each participant was responsible for the initial choice of a cup twice across ten experimental trials. Hereby, the overall influence of regret, which is known to prohibit participants from making switch choices (Stibel et al., 2009), was kept relatively small.

After the MHD trials were completed, each participant was asked to individually complete a three-item questionnaire. First, the participant was asked which strategy (s)he believed was optimal in order to maximize winning chances (i.e., the chance of correctly locating the jolly toy beneath a cup): ‘switching’, ‘staying’, or ‘it does not matter’. Second, the participant was asked to make a posterior probability judgment of winning the prize when staying with the initial cup. Third, the participant was asked to make a posterior probability judgment of winning the prize when switching to the other remaining alternative.

Results

Behavioral Performance on MHD Trials

For each participant, frequencies of switching were counted for the ten experimental trials. Mean frequencies of switching are displayed in Table 2 for each of the nine conditions separately. In line with our hypotheses, switching rates increased when an MHD variant contained a higher number of alternatives. Support for this hypothesis is found for each age group. Next, in the classic version of the MHD with three alternatives, the youngest age group showed a higher switching rate compared to the two oldest age groups. This result is also consistent with our expectations. For the MHD variants with 10 and 50 alternatives, however, the youngest age group showed the lowest switching rate in comparison with the two older age groups.
A two-way analysis of variance was performed with ‘age group’ and ‘number of alternatives’ as independent variables and with switching frequencies as the dependent variable. In line with our hypothesis, results show a statistically significant main effect of ‘number of alternatives’, $F(2, 280) = 222.37, p < .001, \eta^2_{\text{partial}} = .61$. No statistically significant main effect of ‘age group’ was found, $F(2, 280) = 1.35, p = .26, \eta^2_{\text{partial}} = .01$. Next, there is a statistically significant interaction effect between ‘age group’ and ‘number of alternatives’, $F(4, 280) = 6.48, p < .001, \eta^2_{\text{partial}} = .09$. As shown in Table 2, behavioral performance on the MHD improved with increased number of alternatives, but this improvement varied between the age groups. The effect of the number of alternatives is largest in the oldest age group and smallest in the youngest age group: $F(2, 97) = 51.76, p < .001, \eta^2 = .52$ for the primary school participants group, $F(2,95) = 79.79, p < .001, \eta^2 = .63$ for the secondary school participants group, and $F(2, 88) = 113.88, p < .001, \eta^2 = .72$ for the university participants group (all three $p$-values were statistically significant after Bonferroni-Holm correction for multiple testing).

### Understanding of the MHD

After the 10 experimental MHD trials, each participant individually completed a three-item questionnaire in order to assess his/her level of understanding of the problem. In item 1, each participant was asked to indicate which strategy one could best use to maximize winning chances in the MHD (i.e., strategy statement). Switching responses were coded as correct responses, whereas the responses ‘staying’ and ‘it does not matter’ were coded as incorrect. For further analyses, both incorrect responses were merged into one category ‘incorrect’ because only a few participants answered that staying was the optimal strategy.

In item 2, each participant was asked to estimate the probability of winning the prize when staying (i.e., a posterior probability judgment about winning when staying). The responses 33.3%, 10%, and 2% were coded as correct in the 3, 10, and 50 alternatives conditions respectively, whereas other answers were coded as being incorrect.

In item 3, each participant was asked to make a probability judgment for winning when switching (i.e., a posterior probability judgment about winning when switching). The responses 66.7%, 90%, and 98% were coded as correct in the 3, 10, and 50 alternatives conditions respectively. Other answers were coded as incorrect.

Table 3 provides an overview of the percentages of correct responses on the three items of the questionnaire for each condition. This table shows that with increased number of alternatives, the understanding of the MHD seemed to improve within each age group. Next and in line with our hypothesis, for each MHD variant, the youngest participants showed the least understanding of the problem.

Fisher-Freeman-Halton Tests (FFHT’s) were used to test for statistically significant differences in correct answers for the three items between the age groups and between the number of alternatives. A Bonferroni-Holm correction was used to control the Familywise Error Rate at 5%.

First, FFHT’s were performed for each item with ‘age group’ as grouping variable and ‘frequencies of correct and incorrect responses’ as outcome variable, blocked per MHD variant. The results of these tests are displayed in Table 4 and reveal that age group, to some extent, is systematically related to understanding of the MHD variants with 10 (items 1 and 2) and 50 (item 2) alternatives. However, in these MHD variants, age group is not systematically related to performance on the ‘posterior winning probability estimation when switching’ question (item 3). Furthermore, in the MHD variant with 50 alternatives, age group is not systematically related to performance on the strategy statement (item 1). Note that the latter might be explained by a ceiling effect: Table 3 shows that almost all participants assigned to the MHD variant with 50 alternatives correctly indicated switching as the optimal strategy in order to maximize winning chances.

Second, FFHT’s were performed for each item with ‘number of alternatives’ as grouping variable and ‘frequencies of correct and incorrect responses’ as outcome variable, blocked per age group. The results are summarized in Table 5 and show that increasing the number of alternatives seemed to affect understanding of the MHD, especially for the participants of the two oldest age groups. When looking at the results of Table 3 and Table 5 simultaneously, it becomes clear that participants of the two

<table>
<thead>
<tr>
<th>Number of alternatives</th>
<th>Primary school</th>
<th>Secondary school</th>
<th>University</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.52 (0.29)</td>
<td>4.06 (0.29)</td>
<td>3.48 (0.29)</td>
</tr>
<tr>
<td>10</td>
<td>5.82 (0.28)</td>
<td>6.55 (0.30)</td>
<td>7.71 (0.29)</td>
</tr>
<tr>
<td>50</td>
<td>8.79 (0.29)</td>
<td>9.05 (0.27)</td>
<td>9.10 (0.30)</td>
</tr>
</tbody>
</table>

### Table 2: Mean frequencies of switching across 10 MHD trials (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Number of alternatives</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>18.2</td>
<td>31.2</td>
<td>29.0</td>
</tr>
<tr>
<td>10</td>
<td>7.7</td>
<td>26.7</td>
<td>19.4</td>
</tr>
<tr>
<td>50</td>
<td>0.0</td>
<td>3.3</td>
<td>10.0</td>
</tr>
</tbody>
</table>

### Table 3: Percentages of correct answers on the three items of the questionnaire

<table>
<thead>
<tr>
<th>Number of alternatives</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>20.0</td>
<td>50.0</td>
<td>25.0</td>
</tr>
<tr>
<td>10</td>
<td>5.0</td>
<td>22.2</td>
<td>29.2</td>
</tr>
<tr>
<td>50</td>
<td>90.9</td>
<td>100.0</td>
<td>93.1</td>
</tr>
</tbody>
</table>
oldest age groups experienced more advantage of an MHD variant with increased number of alternatives in order to understand the problem compared to participants of the youngest age group. While performance on all three understanding items improved with increased number of alternatives for the secondary school participants, university participants improved only on the strategy statement item and the posterior winning probability judgment when staying, and the youngest age group only improved on the strategy statement item. Increasing the number of alternatives did not help this youngest group of participants to provide more correct posterior probability judgments.

Overall, of the participants who completed all three items ($n = 252$), only 19.0% answered all three correctly and thus showed full understanding of the MHD. The percentages of primary school participants who showed full MHD understanding are 0.0%, 9.7%, and 20.0% in the 3, 10, and 50 alternatives conditions respectively. For the secondary school participants, these percentages equal 3.3%, 22.2%, and 50.0%, whereas for the university participants, these percentages are 10.0%, 29.2%, and 25.0%.

Table 4: P-values of Fisher-Freeman-Halton Tests (blocked per MHD variant) with ‘age group’ as grouping variable

<table>
<thead>
<tr>
<th>Number of alternatives</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>.437</td>
<td>.184</td>
<td>.091</td>
</tr>
<tr>
<td>10</td>
<td>&lt; .001*</td>
<td>.001*</td>
<td>.319</td>
</tr>
<tr>
<td>50</td>
<td>.187</td>
<td>&lt; .001*</td>
<td>.038</td>
</tr>
</tbody>
</table>

Note. * = Statistically significant p-value after Bonferroni-Holm correction for multiple testing.

Discussion

The present study focused on the counterintuitive MHD. Previous MHD studies showed that participants’ tendency to stay with the initial choice is malleable (e.g., Franco-Watkins et al., 2003; Sibiel et al., 2003). In the current paper, we tried to shed more light on this dissociation between behavioral performance and understanding of the MHD. More specifically, we investigated the equiprobability bias (Lecoutre, 1992) in participants of different age groups when confronted with the MHD. In line with previous research, we hypothesized that older participants would be more affected by the equiprobability bias compared to younger participants (see De Neys, 2007). Furthermore, we hypothesized that increasing the number of alternatives in the MHD would weaken the effect of the equiprobability bias, because both the difference between the prior probabilities to initially pick the correct versus wrong door, and the difference between the posterior winning probabilities when staying versus switching, would become more salient.

Considering the behavioral MHD performances, the present findings indicate that when the number of alternatives in the MHD increased, switching rates increased within each age group. This finding can be explained by the fact that increasing the number of alternatives in the MHD makes it very salient that the initial choice is most likely incorrect, therefore eliciting a different reasoning about staying or switching in the second stage, and thus reducing the effect of the equiprobability bias. Next, the youngest age group switched most often in the classic MHD. This finding can be explained by the stronger equiprobability bias with increasing age (De Neys, 2007) and formal statistical education (Morsanyi et al., 2009). Moreover, the influence of regret was minimized in the present study, ruling out the tendency to stay with the initial choice due to anticipation of regret.

Considering MHD understanding, the youngest age group performed worst, which can be easily understood by their generally lower comprehension of probabilistic situations (Fischbein & Schnarch, 1997). In the classic MHD with three alternatives, full problem understanding remained rather low for each age group. For the MHD variants with more than three alternatives, level of understanding differed for the three age groups: Older participants gained a deeper understanding of the problem compared to participants of the youngest age group, which was especially reflected in the answers on the ‘posterior winning probability estimation when staying’ question (item 2). Again, this result can be understood because older participants’ understanding of probabilistic situations is at least minimally developed (Fischbein & Schnarch, 1997). Next, the results on the ‘posterior winning probability estimation when switching’ question (item 3) are notable, because age group did not affect the number of correct answers. This item seems to be extremely difficult, even for participants who have at least minimal understanding of probabilistic situations. Further research should focus on why the ‘posterior winning probability estimation when switching’ question (item 3) is so much harder for participants compared to the ‘posterior winning probability estimation when staying’ question (item 2). This remarkable result is a novel finding, not being...
reported yet in the literature.

When looking at the percentage of participants who showed full understanding the MHD, participants assigned to the MHD with the highest number of alternatives (i.e., 50) performed best. This again can be explained by a smaller effect of the equiprobability bias (cf. supra). Interestingly, secondary school participants outperform university participants in this treatment condition. This finding provides further evidence for a stronger equiprobability bias with increasing age (De Neys, 2007) and statistical education (Morsanyi et al., 2009). Apparently, at an adult age, this bias is developed so strongly that it sometimes prevents understanding, although their understanding of probabilistic situations is at least minimally developed. Notice that secondary school participants outperforming university participants is especially due to the ‘posterior winning probability estimation when switching’ question (item 3), which is much better answered by the secondary school participants.

Given the important role of the equiprobability bias, the dual process theory of reasoning (DPT, see Evans, 2003; Evans & Curtis-Holmes, 2005) may provide a useful framework to understand the findings of the present study: The equiprobability bias, which arises when people rely on the number-of-cases heuristic (Falk, 1992; Shimojo & Ichikawa, 1989), hinders participants to develop a full understanding of the MHD and its underlying probabilities. In order to arrive at a full problem understanding, one should overrule erroneous heuristic reasoning and involve in correct computation. However, our results also provide further critique on a naive conception of the DPT which would state that heuristic reasoning will be replaced by analytic reasoning when cognitive development increases and thus all utterances of heuristic reasoning should decrease with age: The findings of the present study confirm that there exist certain heuristics and biases which strengthen with increasing age (Fishbein & Schnarch, 1997) and statistical education (Morsanyi et al., 2009).

Summarized, our study findings support the previously documented dissociation between behavioral performance and understanding of the MHD (see Franco-Watkins et al., 2003; Slombeck & Tyran, 2004; Stibell et al., 2009; Tubau & Alonso, 2003). However, the nature of this dissociation depends on the participants’ age, and can be decreased for some participants by increasing the number of alternatives.

Acknowledgments

This research was carried out with the financial support of the Concerted Research Action ‘Number sense: Analysis and Improvement’ of the KU Leuven (GOA/12/010).

References


