Effects of Problem Schema on Successful Maximizing in Repeated Choices

Jie Gao (jg2902@columbia.edu)
Center for Decision Sciences, Columbia University
New York, NY 10027 USA

James E. Corter (jec34@columbia.edu)
Teachers College, Columbia University
New York, NY 10027 USA

Abstract
We investigate the effects of problem schema type (complementary events versus independent events) on participants’ tendency to adopt probability matching or maximizing strategies across repeated decisions. These two general problem types were compared in an online study (N=300), using a between-subjects design. We also varied abstraction level of the problem story context, using abstract contexts, contexts involving physical randomizing devices, and “real-world” social/pragmatic contexts. Participants made a binary choice on each of 20 trials, receiving trial-by-trial outcome feedback. Maximization was consistently higher for independent events contexts than for complementary, while abstraction level of the context had no significant effect on the prevalence of maximizing behavior. The results support our hypothesis that people may find it especially difficult to discover the maximizing strategy for problems exemplifying the complementary-outcomes schema. In contrast, when the problem involves choosing between two distinct objects or entities (a common instantiation of the independent events schema) it seems to be easier to maximize, perhaps due to cuing of a pragmatic “pick the winner” schema.

Keywords: probability matching; maximizing; repeated decisions; pragmatic schemas; abstraction

Introduction
It has been shown that past experience has an important impact on people’s reasoning and decision making. In one influential line of research, Cheng and Holyoak (1985) argue that people often reason not according to the rules of formal logic, but based on a set of abstract (or partly abstract) knowledge structures induced from daily-life experience. They term these knowledge structures “pragmatic reasoning schemas”. These pragmatic schemas have been shown to affect people’s perceptions, problem solving and decision making. For example, many studies have shown that reasoning problems that are situated in a real-world context are solved more easily than those posed in purely abstract forms (Evans, 1982; Johnson-Laird et al., 1972; Wason, 1966; Wason & Evans, 1975). Other researchers have found a significant impact of previous life experience on perception and cognition, both experience in the physical world (Tooby & Cosmides, 1992) and in domains that are abstracted from or unrelated to the physical environment (cf. Bargh, 2006). Williams, Huang and Bargh (2009) have used the term “mind scaffolding” to refer to how higher mental processes are often grounded in early experience of the physical world.

Regarding statistical concepts, Nisbett, Krantz, Jepson, and Kunda (1983) argue that people learn intuitive versions of abstract principles such as the law of large numbers through life experience in various domains. These representations are often referred to as statistical heuristics. These heuristics can be improved by statistical training and successfully applied across domains (Fong, Krantz & Nisbett, 1986). Nonetheless, people do not always use statistical reasoning when it is appropriate (e.g., Tversky & Kahneman, 1974). Accordingly, Nisbett et al. argue that it is important to study what kinds of events and problems most often elicit statistical reasoning. They list three factors that affect whether statistical reasoning is applied in a particular context: the clarity of the sample space and the sampling process, the perceived relevance of chance factors, and cultural norms for the specific domain.

We believe that such pragmatic or semantic factors can affect learning and application of formal knowledge across a broad array of tasks, including optimal versus non-optimal choices in decision making. Thus, in the present paper we investigate the influence of such prior pragmatic knowledge on people’s ability to find and select the “rational” or optimal strategy in repeated random binary choices, a situation where sub-optimal behavior (specifically, probability matching) is not infrequent.

To illustrate, imagine Situation 1, wherein a die with two faces colored black and four colored red is rolled, and suppose that your task is to predict the color that will occur on each of the next five rolls. Now imagine Situation 2, where there are two dice, the first colored as above and the second with the colors reversed. Here, your task is to predict which die will show red, on each of five successive trials.

Although the two situations share many surface features (and the same objective probability of success on each trial, if you pick the more likely outcome), we believe that they are psychologically different, that they are associated with different experience-based pragmatic schema. In the first example, the two possible outcomes, black and red, are not just negatively correlated, they are complementary events. In terms of experience, this situation may be associated with common everyday examples of repeatedly trying to predict a binary outcome: Heads versus Tails, making a shot in basketball versus missing it, will it be a sunny day or a rainy day? In most of these experienced situations, the element of chance variation is very salient.
In the second situation, the two events (Die 1 shows red) and (Die 2 shows red) are not correlated, rather they are independent events. The decision maker is in the position of trying to select an entity that will offer superior outcomes. This type of situation may recall experiences with “pick a winner” type scenarios: which player is more likely to hit a free throw, which berry bush will have more berries on it? In many such real-life situations, causal models might be invoked, correctly or incorrectly.

Note that Situation 1 resembles tasks commonly used in studies of probability matching (e.g., James & Koechner, 2011; Shanks, 2002; Barron & Leider, 2010), a well-documented form of suboptimal behavior. The literature on probability matching amply documents that people often fail to use maximizing strategies when making sequential predictions (e.g., Chen & Corter 2006; Hertwig, Barron, Weber, & Erev, 2004). Many of these studies have used tasks where participants repeatedly predict outcomes for simple randomizing devices: coins or dice. But do the documented failures to maximize generalize beyond this narrow situation? Or would people do better in an independent-events scenario?

We hypothesize that the answer to the latter question is affirmative. Put another way, we suspect that the failures to maximize commonly observed in the probability-matching literature may depend critically on the complementary-outcomes nature of the prediction task. In a coin flip, or any type of process with complementary-outcomes, observing that Heads has occurred means that Tails has not. For such events, most people have vast prior experience observing mixed sequences of outcomes, as well as an appreciation of the futility of trying to predict the next flip. These experiences may impede their ability to find the maximizing strategy if one exists (for a biased coin that strategy involves predicting pure sequences of the more likely outcome).

In contrast, for the independent-events situation, a participant does not predict the outcome of one repeated (identical) event with binary outcomes, rather s/he is choosing between two different and independent options, where the options may represent distinct entities. In real life, it is common for people to expend a great deal of effort to choose the better option (entity). For example, people try to choose a better partner to work with for every project they do, or to choose a better mate, or to choose a better problem-solving strategy, etc. We term the pragmatic schema formed from these types of life experience the “pick a winner” schema.

For the above reasons, we hypothesize that decisions and choices involving independent events will result in more optimal behavior than those involving complementary events, even when the payoff probabilities for the options in the two situations are the same, because two different schemas are likely to be activated: the complementary-events schema which leads to probability matching and the “pick-a-winner” independent events schema which is more likely to lead to maximizing behavior.

The independent events (pick-a-winner) schema is an abstraction, as is the complementary-events schema. However, we believe that these particular abstractions arise from experience, thus they retain some associations with semantic aspects of real-world situations – i.e. that they are “pragmatic schemas” (Cheng & Holyoak, 1985; Marshall, 1993; Nisbett 1993; Nisbett, Krantz, Jepson, & Kunda, 1983). Many researchers (e.g., Bassok, Chase, & Martin, 1998; Reeves & Weisberg, 1994) have demonstrated that formal reasoning methods may be more accurately used in an applied context, due to the effects of experience-based semantic knowledge. Bassok and co-workers argue that the applied or “grounded” context must map onto the abstract schema in order for people to better understand or apply the reasoning rules.

If reasoning problems situated in a real-world context tend to be solved more easily than those posed in purely abstract forms (Evans, 1982; Johnson-Laird et al., 1972; Wason, 1966; Wason & Evans, 1975), another factor that may influence the prevalence of maximizing behavior is level of abstraction of the problem context. Accordingly, in the present study we also investigate if people do better at identifying and applying the normative strategy with abstract scenarios (representing “fully abstracted” schemas for complementary and independent events), or with “grounded” story contexts involving concrete objects or other real-world situations (that should invoke pragmatic experiential knowledge). The real-world problems are of two types: “concrete” problems involving simple randomizing devices (e.g. coins, dice) that ground the decision problems in the behavior of simple physical objects, and pragmatic or social problems (games, foraging) that might be influenced by these sorts of experiences.

**Empirical Study**

This study is designed to investigate factors influencing people’s ability to find and apply the normative “maximizing” strategy in a repeated-decisions task. To investigate the role of pragmatic schema in this task, we compared people’s performance on problems exemplifying the two schema types (independent events vs. complementary events), including scenarios with three different abstraction levels: abstract, concrete physical (randomizing devices, such as dice), and “real-world” pragmatic or social contexts. Thus, there are six conditions, as shown in Table 1. In order to increase generalizability of the findings, we used more than one example of “concrete” and “real-world” problem types.

<table>
<thead>
<tr>
<th>Table 1. Conditions and Scenarios</th>
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<tbody>
<tr>
<td>Complementary</td>
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<tr>
<td>Abstract</td>
</tr>
<tr>
<td>Concrete</td>
</tr>
<tr>
<td>Real-world</td>
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</table>
We hypothesize that in the independent-schema scenarios people will be more likely to choose the optimal decision alternative than in the complementary events. We have no specific hypothesis about the effect of level of abstraction of the problem contexts, although based on the previous literature on grounding and application of formal schemas one might expect performance to be best in the concrete contexts.

Materials

In designing the materials, we took care to make sure that the descriptions of the problems were as parallel as possible between scenarios and schema types. This may be checked by examining the following example problems for each of the six conditions. For instance, comparing example 1 (Complementary Events, Abstract context) with example 4 (Independent events, Abstract context), both problems offer a choice between an option A that has a 2/3 chance of “success”, and an option B that offers 1/3 chance, with no further information other than the contrasting schemas.

1. Complementary Events, Abstract context:
   You will be asked to predict 20 trials of a random event. On each trial you will choose one of two options, A or B. Then, either Event A occurs (with probability 2/3) or Event B occurs. If the event you predicted occurs, that trial is a “success”.

2. Complementary Events, Concrete context (dice):
   You will be asked to predict 20 trials of a random event, the color that shows when a single die is rolled. The die has 4 of its faces colored Red, and 2 faces colored White. On each trial you will pick either Red or White, then the die is rolled. If the face of the die that comes up shows the color you picked, that trial is a “success”.

3. Complementary Events, “Real-World” context (basketball):
   You will be asked to predict hits and misses as a basketball player, Player X, shoots 20 free throws. Player X hits 2/3 of his free throws on average. On each trial you will predict Hit or Miss. If you predict correctly, that trial is a “success”.

4. Independent Events, Abstract context:
   You will be asked to predict 20 trials of a random event. On each trial, you will choose one of two options, A or B. On any trial, Option A has a 2/3 chance of showing a “success”. Independently, on any trial Option B has a 1/3 chance of showing a “success”. Your goal is to choose one that shows a “success”.

5. Independent Events, Concrete context (dice):
   You will be asked to predict 20 trials of a random event, whether a die with colored faces shows a Red face when it is rolled. There are two dice, Die A and Die B. Die A has 4 of its faces colored Red, and 2 faces colored White. Die B has 2 of its faces colored Red, and 4 faces colored White. On each trial you will pick one of the two dice. If the face of the die you picked shows Red, that trial is a “success”.

   You are participating in a basketball free-throw shooting contest. Your team consists of two players, A and B. Player A hits 2/3 of his free throws on average, while Player B hits 1/3 of his free throws on average. On each trial, you get to select a player to take a single free throw. If the player you select hits that free throw, that trial is a “success”.

Participants

Participants were recruited from an online worker marketplace, Amazon's Mechanical Turk. We specified in our instructions that each participant can only do this task once, and subsequent completions would not be accepted.

We recruited 25 participants for each specific problem, so that the total sample size was 300. Participants were restricted to US location only. The average age was 29.31, ranging from 13 to 68. 38.7% of them were female, and 61% were male. Only 4.6% of them had taken more than three statistics or mathematics courses at the college level, 45.6% had taken one to three courses, and the rest of them had not taken any.

Procedure

The task took about 5 minutes to complete. Participants were paid $0.25 as the base pay for completing the study, plus a performance-based bonus payment (in the amount of $0.05 for each of the trials that they predict correctly). The expected total payment for a person using maximizing strategy is $1.05 (0.25+0.05*80%*20), and for someone using a probability matching strategy the expected total payment is $0.93 (0.25+0.05*68%*20).

Upon accepting the task, participants were randomly directed to one of the 12 specific-scenario conditions. After reading the instructions, they were asked to complete the prediction task twenty times (trials). They made their choice for each trial by clicking one of two side-by-side buttons on the screen. After they made a choice for a trial, they were shown immediate feedback (randomly generated according to the specified probabilities). Feedback consisted of the actual obtained outcome, plus notification of whether their prediction or choice was a “success” (i.e., paid off) in the current trial. The feedback screen also displayed the accumulated amount of bonus payment they had won. Afterwards a questionnaire was given to each participant, asking about the strategies he or she used and basic demographic information such as gender, age, math ability and educational background.

Results

The probability of choosing the optimal alternative was compared across conditions. Prediction accuracy (the prevalence of maximizing behavior) was measured by coding a response 1 if the participant made the “rational” maximizing choice (i.e., chose the outcome with the higher probability of payoff), and 0 for the other choice.

The mean probabilities of choosing the more likely outcome for each specific problem are shown in Table 2. These means are computed across both trials and
participants. Overall, less maximizing behavior was observed for complementary events (Mean proportion = .778, SD = .016) than for independent events (Mean = .835, SD = .014).

Table 2 Probability of predicting the more likely outcome: mean and standard error for each specific context, by condition (N=25 for all cells).

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Complementary</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract (A or B)</td>
<td>0.75 (0.051)</td>
<td>0.83 (0.038)</td>
</tr>
<tr>
<td>Dice</td>
<td>0.83 (0.037)</td>
<td>0.86 (0.035)</td>
</tr>
<tr>
<td>Marbles</td>
<td>0.83 (0.037)</td>
<td>0.82 (0.032)</td>
</tr>
<tr>
<td>Spinners</td>
<td>0.71 (0.044)</td>
<td>0.80 (0.042)</td>
</tr>
<tr>
<td>Basketball Players</td>
<td>0.79 (0.029)</td>
<td>0.83 (0.036)</td>
</tr>
<tr>
<td>Fishing</td>
<td>0.76 (0.033)</td>
<td>0.87 (0.031)</td>
</tr>
<tr>
<td>Total</td>
<td>0.77 (0.016)</td>
<td>0.84 (0.015)</td>
</tr>
</tbody>
</table>

In order to test the hypotheses concerning the effects of schema type and the three abstraction levels on selection of the more likely option, the data were conceptualized as a 2 x 3 unbalanced ANOVA design. Because the data for each subject consist of 20 binary variables (the choice on each trial) for a given problem, the data were analyzed as a generalized linear mixed model (GLMM), using the GLIMMIX function in SPSS. The model included main effects for abstraction level, schema type, and trial, plus all two-way interactions. The covariance structure was assumed to be AR(1), i.e., that the response at time point \(t+1\) is correlated with the response at time point \(t\).

The results show that the main effect of schema type is significant, \(F(1, 5918) = 20.418, p < .001\). The direction of this effect is consistent with our expectation that independent events contexts should lead to more maximizing behavior because of the “pick a winner” schema, which makes it seem more reasonable to predict the alternative with higher payoff probability for all the trials. The main effect of schema type was consistently found across different abstraction levels of the scenarios. As shown in Figure 1, there was no significant interaction effect with abstraction level of context (abstract, concrete, real-world), \(F(2, 5918)=1.396 \text{ p = .248}\), and independent events led to more choice of the optimal strategy for all three context levels.

The mean probability of choosing the more likely outcome did not differ greatly across abstraction levels. Descriptive statistics were as follows: abstract context (M = .791, SD = .027), concrete context (M = .807, SD = .015), and the real-world context (M = .813, SD = .019).

The results of a generalized linear mixed model (GLMM) analysis showed that the main effect of abstraction level was not significant, \(F(2, 5918) = .475, \text{ p = .62}\). This result suggests that the probability of choosing the more likely outcome is not affected by the abstraction level of a task, i.e., whether it is abstract, involves concrete random devices, or “real-world” social or pragmatic contexts.

**Trend over trials – effects of DM experience**

In repeated decision tasks with outcome feedback, experience can change the proportions of maximizing behavior (e.g., Chen & Corter, 2014; Yechiam & Busemeyer, 2005; Newell & Rakow, 2007). In order to assess if there was any change in maximizing responses across trials here, the twenty trials were divided into four “pseudo-blocks” or pseudo-sets of five trials each. Figure 2 shows the mean probability of choosing the optimal alternative across four pseudo-sets. The graph shows that the advantage in maximizing behavior for the independent events scenarios over the complementary events scenarios is relatively stable across trials, and the overall levels of maximization did not show any consistent linear trend. The dip in maximization in the middle pseudo-blocks may be due to exploratory behavior.

![Figure 1. Probability of choosing the more likely outcome (Schema Type x Abstraction Level)](image1.png)

![Figure 2. Mean probability of maximizing for the two schema types across blocks.](image2.png)
Discussion

The results supported our hypothesis that problems exemplifying the complementary-outcomes schema would lead to a lower rate of maximization than problems exemplifying the independent-events schema.

Differences between these two schema types in the proportion of “rational” maximizing responses (i.e., selection of the more likely outcome) were consistently observed across the three different abstraction levels of event contexts, and no significant interactions were found. Thus, the main effect of complementary- versus independent-events schema generalized across different abstraction levels of context: in abstract, concrete or pragmatic “real-world” contexts. We included more than one random device and more than one real-world context, and the effect was observed in all contexts. Thus, the results demonstrate that the type of schema, complementary versus independent events, has a strong and consistent effect on prediction behavior for stochastic binary events.

The results shed light on cognitive factors that affect why, and when, people are successful in finding maximizing strategies. We suspect that a primary cause of this gap between the rate of maximizing for independent and complementary events is that when people must choose between two objects or entities (a common instantiation of the independent events schema), they will be more prone to maximize due to influence of the “pick a winner” pragmatic schema.

We further believe that people’s difficulty in finding the maximizing strategy for complementary events arises due to prior experience (and is related to a representativeness heuristic) – For a coin flip, it is natural to envision mixed sequences, but less natural to envision a sequence of all Heads or Tails, because there are probabilistic constraints that make “pure” sequences quite rare in one’s experience. Yet for biased events with one outcome more likely than the other, such pure sequences correspond to the optimal maximizing choice strategy. In contrast, for “pick a winner” situations, expert performance might often result in perfect performance over many trials (imagine a professional violinist playing a well-learned etude). These arguments suggest why probability-matching behavior, a well-known type of decision irrationality, may not generalize robustly beyond tasks based on complementary events.

Additionally, it may be that people’s predilection to see patterns in randomness and to search for causal interpretations for these patterns plays a role in differential behavior and cognition in situations exemplifying the two schemas. For example, it may be hard to generate causal explanations (other than simple physical principles) for why a coin lands Heads rather than Tails, but easier when choosing among fishing ponds or basketball players. Thus, the present findings suggest that such causal beliefs might have adaptive impact even when erroneous. However, Ayeton and Fisher (2004) document that inappropriate or irrational expectations of recency effects in repeated events, as exemplified by the “hot-hand” fallacy in basketball (Gilovich, Vallone & Tversky, 1985) and the Gambler’s Fallacy, also seem to be influenced by problem context (e.g., human performance versus chance processes). Thus, irrational behaviors as well as rational may be influenced by problem context and problem schema.

In the present study, abstraction level of the problem context was not found to have a significant effect on the prevalence of rational choice. This is somewhat surprising given the body of prior research that shows advantages in formal reasoning for concrete as opposed to abstract contexts. One possible explanation is that in our study the repeated choice task may be more influenced by experience- or intuition-based decision processes than by formal reasoning. Another possibility worth investigating is that some of this prior research might have confounded schema type, as defined here, and abstraction level.

A seemingly attractive alternative explanation for our results is that the complementary events and independent events situations offer different amounts of information about foregone payoffs, a factor that has been shown to affect tendency to maximize in some repeated decision tasks (e.g., Yechiam & Busemeyer, 2006). Here, the occurrence of one event in a pair of complementary events (such as Heads) provides certainty that the complementary event (Tails) did not occur, whereas independent events by definition offer no predictive information about each other.

However, information about foregone payoffs cannot account for the findings in this study. First, in previous research foregone payoff information was found to be helpful in prediction tasks when the payoff probabilities were initially unknown to the participants (e.g., Yechiam & Busemeyer, 2006; Grosskopf, Erev, & Yechiam, 2006). In our study, there is no probability learning – the probabilities of the two events were available from the first trial. Second, the usual effect of providing information on foregone payoffs is to speed up learning of the optimal response. But in our task, it is the complementary-events condition that inherently provides information on foregone payoffs, yet this condition led to a lower rate of maximization here. Thus, availability of information on foregone payoffs cannot account for our results.

Elsewhere (Gao, 2013; Gao and Corter, 2011) we have proposed that a decision maker’s implicit adoption of a perfect prediction goal might be a factor promoting probability matching behavior. That factor might also be playing a role in this study. For complementary events, the choice is between "either A or B", thus there is always a "correct/successful" answer and a "wrong/unsuccesful" answer; while for independent events, the choice is between "better A? or better B?", where it is more apparent that neither of the two options is guaranteed to lead to a successful result. Therefore, the independent events scenario might nudge people to abandon any unrealistic perfect prediction goal, hence lead to more rational choices. However, this potential explanation is speculative, and would need to be explored in future studies.
References


