Modeling Gain-Loss Asymmetries in Risky Choice: The Critical Role of Probability Weighting

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Abstract

A robust empirical regularity in decision making is that the negative consequences of an option (i.e., losses) often have a stronger impact on people’s behavior than the positive consequences (i.e., gains). One common explanation for such a gain-loss asymmetry is loss aversion. To model loss aversion in risky decisions, prospect theory (Kahneman & Tversky, 1979) assumes a kinked value function (which translates objective consequences into subjective utilities), with a steeper curvature for losses than for gains. We highlight, however, that the prospect theory framework offers many alternative ways to model gain-loss asymmetries (e.g., via the weighting function, which translates objective probabilities into subjective decision weights; or via the choice rule). Our goal is to systematically test these alternative models against each other. In a reanalysis of data by Glöckner and Pachur (2012), we show that people’s risky decisions are best accounted for by a version of prospect theory that has a more elevated weighting function for losses than for gains but the same value function for both domains. These results contradict the common assumption that a kinked value function is necessary to model risky choices and point to the neglected role of people’s differential probability weighting in the gain and loss domains.

Keywords: cognitive modeling; loss aversion; risky choice; prospect theory; probability weighting

Introduction

For many of our decisions we are unable to tell with certainty what consequence the decision will have—for instance, when deciding between different medications that potentially lead to some side effects. Ideally, we have knowledge of the nature of the possible consequences as well as some inkling of the chances that the consequences will occur, but our decisions must necessarily remain in the “twilight of probability” (Locke, 1690/2004). Elaborating how such risky decisions are made (and how they should be made) has engaged decision scientists at least since Bernoulli’s (1738/1954) seminal work on subjective utility.

One of the most influential and successful modeling frameworks of risky decision making is prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). A prominent feature of prospect theory is the assumption that the subjective disutility of a negative outcome is higher than the subjective utility of a positive outcome of the same size. In other words, prospect theory assumes an asymmetry between gains and losses in its value function, which translates objective outcomes into subjective magnitudes. This assumption of loss aversion can explain, for instance, that people dislike gambles in which one has a 50% chance to win a particular amount of money and a 50% can to lose the same amount. Similarly, loss aversion is invoked to account for the endowment effect—the phenomenon that people evaluate an object higher in a buyer perspective than in a seller perspective (e.g., Pachur & Scheibehenne, 2012; for a general overview of gain-loss asymmetries, see Peeters & Czapinski, 1990).

However, the way prospect theory—more specifically, its mathematical formulation in cumulative prospect theory (CPT; Tversky & Kahneman, 1992)—is usually implemented allows for asymmetries in the evaluation of positive and negative prospects to be represented also in other ways than via the value function. For instance, the parameters of CPT’s weighting function, which translates objective probabilities into subjective decision weights, are typically estimated separately for the gain and the loss domain (e.g., Gonzalez & Wu, 1999). Furthermore, it has been argued that choice sensitivity (i.e., how accurately choices between two alternatives reflect their subjective valuations) differs between options involving losses and those involving gains only (Yechiam & Hochman, 2013a).

Crucially, these possible representations of gain-loss asymmetries within CPT have never been directly pitted against each other in a model-comparison analysis (Linhart & Zucchini, 1986), where the descriptive power of a model is evaluated in light of its complexity (but see Harless & Camerer, 1994; Stott, 2006). Conducting such a model comparison is our goal in this paper. To that end, we use CPT to model data collected by Glöckner and Pachur (2012), where 64 participants were asked to make choices between 138 two-outcome monetary gamble problems.1 Fitting different implementations of CPT to this data also allows us to test specific predictions of how a gain-loss asymmetry should be reflected in specific parameter patterns, such as choice sensitivity (Yechiam & Hochman, 2013a) or probability sensitivity (Wu & Markle, 2008). Next we provide a detailed description of CPT’s parameter

1 In Glöckner and Pachur (2012) each participant made choices between 138 gamble problems at two separate sessions (separated by one week). Here we analyze the data from the first session.
framework, which we then use to formalize different ways to represent gain-loss asymmetries in risky decision making.

Cumulative Prospect Theory
According to CPT, the possible consequences of a risky option are perceived as gains or losses relative to a reference point. The overall subjective value $V$ of an option with outcomes $x_0 > \ldots > x_1 > 0 > y_1 > \ldots > y_n$ and corresponding probabilities $p_m, \ldots, p_1; q_1, \ldots, q_n$ is given by:

$$V(A) = \sum_{i=1}^{n} v(x_i)p_i^\gamma + \sum_{j=1}^{n} v(y_j)q_j^{-\delta},$$

where $v$ is a value function satisfying $v(0) = 0$; $\pi^+$ and $\pi^-$ are decision weights for gains and losses, respectively, which result from a rank-dependent transformation of the outcomes' probabilities. The decision weights are defined as:

$$\pi^+_i = w^+(p_i)$$
$$\pi^-_i = w^-(q_i)$$
$$\pi^+_i = w^+(p_i + \ldots + p_m) - w^+(p_{i+1} + \ldots + p_n) \quad \text{for} \quad 1 \leq i < m$$
$$\pi^-_j = w^-(q_j + \ldots + q_m) - w^-(q_{j+1} + \ldots + q_n) \quad \text{for} \quad 1 \leq j < n,$$

with $w^+$ and $w^-$ being the probability weighting function for gains and losses, respectively (see below). The weight for each positive outcome represents the marginal contribution of the outcome's probability to the total probability of obtaining a better outcome; the weight for each negative outcome represents the marginal contribution of the outcome's probability of obtaining a worse outcome.

Several functional forms of the value and weighting functions have been proposed (see Strotz, 2006, for an overview). In our analyses, we use the power value function suggested by Tversky and Kahneman (1992), which is defined as

$$v(x) = x^\alpha$$
$$v(y) = -\lambda(-y)^\beta.$$

For $\alpha$ and $\beta$ usually values smaller than 1 are found, yielding a concave value function for gains and a convex value function for losses. The parameter $\beta$ reflects the relative sensitivity to losses versus gains and is often found to be larger than 1, indicating loss aversion.

The weighting function has an inverse S-shaped curvature, indicating overweighting of unlikely events (i.e., those with a small probability) and underweighting of likely events (i.e., those with a moderate to high probability). Here we use a two-parameter weighting function originally proposed by Goldstein and Einhorn (1987), which separates the curvature of the function from its elevation (cf. Gonzalez & Wu, 1999):

$$w^+(p) = \frac{\delta^+ p^\gamma}{\delta^+ p^\gamma + (1-p)^\gamma} \quad \text{for} \quad x,$$
$$w^-(q) = \frac{\delta^- q^\gamma}{\delta^- q^\gamma + (1-q)^\gamma} \quad \text{for} \quad y,$$

$\gamma^+$ and $\gamma^-$ (both $< 1$) govern the curvature of the weighting function in the gain and loss domains, respectively, and indicate the sensitivity to probabilities. The parameters $\delta^+$ and $\delta^-$ (both $> 0$) govern the elevation of the weighting function for gains and losses, respectively, and can be interpreted as the attractiveness of gambling. In other words, $\delta^+$ and $\delta^-$ also indicate a person's risk attitude, with higher (lower) values on $\delta^+$ ($\delta^-$) for higher risk aversion in gains (losses).

In addition to these core components of CPT, a choice rule is required when applying CPT to model binary choice. To derive the predicted probability of CPT that a gamble A is preferred over a gamble B we used an exponential version of Luce's choice rule:

$$p(A,B) = \frac{e^{\delta^V(A)}}{e^{\delta^V(A)} + e^{\delta^V(B)}},$$

where $\phi$ is a choice sensitivity parameter, indicating how sensitively the predicted choice probability reacts to differences in the valuation of gambles A and B. A higher $\phi$ indicates more deterministic behavior; with $\phi = 0$, choices are random.

Modeling Gain-Loss Asymmetries
As described in the previous section, the common approach to accommodate an asymmetric evaluation of positive and negative prospects is to assume a kinked utility function, for instance produced by $\lambda > 1$ (see also Usher & McClelland, 2004; Ahn, Busemeyer, Wagenmakers, & Stout, 2008). Note, however, that observed choices are modeled based on three intertwined components, a value function, a weighting function, and a choice rule, all of which could, in principle, represent an asymmetry between gains and losses. In the following, we describe how gain-loss asymmetries could be modeled within each these components.

Utility Accounts
The formalization of CPT's value function allows for two ways to represent a gain-loss asymmetry.

Differential weighting of losses and gains Tversky and Kahneman's (1992) original version of CPT accommodates a gain-loss asymmetry using the loss aversion parameter $\lambda$, with $\lambda > 1$ leading to a stronger impact of losses (relative to gains). As can be seen from Equation 3, the effect of $\lambda$ is to multiplicatively magnify the utility of losses relative to the utility of gains, implying greater sensitivity to losses.

Differences in outcome sensitivity In many applications of CPT the exponent of the value function is estimated separately for gains and losses (cf. Fox & Poldrack, 2008). If the latter (i.e., $\beta$ in Equation 3) is higher than the former (i.e., $\alpha$ in Equation 3), this could also lead to a kinked utility function, and thus a gain-loss asymmetry. Note that this pattern has been observed in studies that included pure gain and pure loss gambles (e.g., Abdellaoui, Vossman, & Weber, 2005).

Probability Weighting Accounts
Equation 1 shows that according to CPT—as in other models in the expectation tradition—the evaluation of an
option closely ties the outcomes to their probabilities, as both are combined multiplicatively. Therefore, an apparent gain-loss asymmetry in the choices might also be represented by assuming differences between gains and losses in probability weighting (Zank, 2010; see also Birnbaum, 2008). Existing studies that have estimated the weighting function separately for gains and losses show that doing so indeed partly absorbs a gain-loss asymmetry (and might decrease the estimated value of $\lambda$). In particular, the elevation is commonly found to be higher for losses than for gains (for an overview, see Fox & Poldrack, 2008). Nevertheless, it is currently unclear to what extent estimating different weighting functions for losses and gains interacts with the estimation of the $\lambda$ parameter and whether the increased model flexibility gained by adding more parameters actually leads to better predictive performance.

Differences in probability sensitivity Wu and Markle (2008) highlighted that an asymmetry might not necessarily exist between gains and losses, but between problems with mixed gambles and problems with single-domain gambles (i.e., those that offer either only gains or only losses). They found support for a version of CPT that allows probability sensitivity to differ between mixed and single-domain problems, with a lower probability sensitivity for mixed gambles than for single-domain problems. Moreover, Wu and Markle showed that this version of CPT can account for differences in probability sensitivity (i.e., the curvature of the weighting function) is lower for mixed gambles than for single-domain gambles. Wu and Markle found support for this pattern using Tversky and Kahneman’s (1992) one-parameter weighting function; one limitation of this function is, however, that curvature and elevation are confounded. Whether the hypothesized parameter pattern also emerges when using a function that allows to disentangle curvature and elevation (e.g., using the two-parameter weighting function described in Equation 4) has not yet been tested.

Choice Sensitivity Account
A radically different explanation for an asymmetry between the gain and the loss domain was proposed by Yechiam and Hochman (2013a). They argued that the somewhat inconsistent manifestation of loss aversion in risky choice studies might be due to the fact that processing information about potential losses increases the amount of attention allocated to the task at hand. According to Yechiam and Hochman, this should be reflected in a higher choice-sensitivity parameter in problems involving losses (i.e., pure-loss gambles and mixed gambles) as compared to problems involving gains only. In a task in which participants responded to sequentially learned risks and using a reinforcement model, Yechiam and Hochman (2013b) found support for this hypothesis; to our knowledge, it has not been tested in the context of description-based tasks and using CPT as modeling framework.

Which Model Provides The Best Account?
Several investigations have challenged the utility account of gain-loss asymmetries (e.g., Schmidt & Traub, 2002; Yechiam & Hochman, 2013a). However, one problem of these studies is that they focused on specific items and are thus silent with regard to the importance of the elements of utility accounts (e.g., the loss aversion parameter) for CPT’s ability to describe risky choices more generally.

For a more general test, one needs to compare different CPT implementations (representing alternative accounts of gain-loss asymmetries) and to determine which fares best in trading off model fit and model complexity (Myung, 2000). Such a modeling analysis also allows us to test hypotheses concerning specific parameter patterns predicted by some of these accounts. For instance, according to the choice-sensitivity account by Yechiam and Hochman (2013a) choice sensitivity should be higher in tasks involving losses than in tasks involving gains only. This hypothesis has not been tested directly in the context of description-based tasks.

A second hypothesized parameter pattern follows from the probability weighting account proposed by Wu and Markle (2008), according to which the probability sensitivity (i.e., the curvature of the weighting function) is lower for mixed gambles than for single-domain gambles. Wu and Markle found support for this pattern using Tversky and Kahneman’s (1992) one-parameter weighting function; one limitation of this function is, however, that curvature and elevation are confounded. Whether the hypothesized parameter pattern also emerges when using a function that allows to disentangle curvature and elevation (e.g., using the two-parameter weighting function described in Equation 4) has not yet been tested.

Modeling Approach
To evaluate the different accounts of gain-loss asymmetries described above, we tested a total of 10 different implementations of CPT in their ability to describe people’s risky choices. The implementations, which are summarized in Table 1, differ in terms of whether a gain-loss asymmetry is represented in the value function, the weighting function, or the choice rule.

<table>
<thead>
<tr>
<th>Model</th>
<th>Outcome sensitivity</th>
<th>Probability sensitivity</th>
<th>Elevation</th>
<th>Loss aversion</th>
<th>Choice sensitivity</th>
<th>Free parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPT\textsubscript{naive}</td>
<td>$\alpha$</td>
<td>$\gamma$</td>
<td>$\delta$</td>
<td>$\lambda=1$</td>
<td>$\phi$</td>
<td>4</td>
</tr>
<tr>
<td>CPT\textsubscript{1}</td>
<td>$\alpha$</td>
<td>$\gamma$</td>
<td>$\delta$</td>
<td>$\lambda$</td>
<td>$\phi$</td>
<td>5</td>
</tr>
<tr>
<td>CPT\textsubscript{ab}</td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\gamma$</td>
<td>$\delta$</td>
<td>$\lambda=1$</td>
<td>5</td>
</tr>
<tr>
<td>CPT\textsubscript{gd}</td>
<td>$\alpha$</td>
<td>$\gamma^+$</td>
<td>$\delta^+$</td>
<td>$\delta^-$</td>
<td>$\lambda$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>CPT\textsubscript{ed}</td>
<td>$\alpha$</td>
<td>$\gamma$</td>
<td>$\delta^+$</td>
<td>$\lambda$</td>
<td>$\phi$</td>
<td>6</td>
</tr>
<tr>
<td>CPT\textsubscript{fix}</td>
<td>$\alpha$</td>
<td>$\gamma$</td>
<td>$\delta$</td>
<td>$\lambda=1$</td>
<td>$\phi$</td>
<td>5</td>
</tr>
<tr>
<td>CPT\textsubscript{fix-p}</td>
<td>$\alpha$</td>
<td>$\gamma^+$</td>
<td>$\delta$</td>
<td>$\lambda$</td>
<td>$\phi$</td>
<td>6</td>
</tr>
<tr>
<td>CPT\textsubscript{phi}</td>
<td>$\alpha$</td>
<td>$\gamma^+$</td>
<td>$\delta$</td>
<td>$\lambda=1$</td>
<td>$\phi^*$</td>
<td>5</td>
</tr>
<tr>
<td>CPT\textsubscript{gum}</td>
<td>$\alpha$</td>
<td>$\gamma^+$</td>
<td>$\delta$</td>
<td>$\lambda$</td>
<td>$\phi$</td>
<td>5</td>
</tr>
</tbody>
</table>

CPT\textsubscript{1} can be considered as the standard implementation of CPT. It assumes the same exponent in the value function
For the standard version of CPT, CPT\(_{\text{std}}\), which allows for a gain-loss asymmetry only through the loss aversion parameter, the median (across participants) best-fitting value of the \(\lambda\) parameter was substantially larger than 1, \(\lambda = 1.40\). 71.9\% of the participants had a \(\lambda\) larger than 1. Moreover, CPT\(_{\text{std}}\) showed a considerably better fit than CPT\(_{\text{loss}}\), which does not allow for an asymmetry between gains and losses (median BIC: 158.34 vs. 160.23). These results thus provide evidence for a gain-loss asymmetry in the data.

**Is choice sensitivity higher in gambles involving losses?**

As previously stated, Yechiam and Hochman (2013a) argue that due to differences in attention, choice sensitivity should be higher when the gambles include a potential loss. We tested this prediction by modeling the data with CPT\(_{\text{phi}}\), which allows for a gain-loss asymmetry in choice sensitivity only. As it turned out, there was no evidence for Yechiam and Hochman’s hypothesis; in fact, we find the opposite pattern, with a higher choice-sensitivity parameter for gains than for losses, median values \(\phi^+ = 0.18\) and \(\phi^- = 0.09\). Wilcoxon test: \(W = 2,609, p = .0008\) (two-tailed). This pattern of results was found for 58 of the 64 participants (91%).

**Is probability sensitivity lower in mixed gambles?**

Consistent with Wu and Markle’s (2008) hypothesis, the estimates for \(\gamma\) obtained with CPT\(_{\text{pm}}\) indicated a lower probability sensitivity for mixed gambles than for single-domain gambles, median values \(\gamma^+=0.58\) and \(\gamma^- = 0.86\). Wilcoxon test: \(W = 2,928, p = .0001\) (two-tailed). Forty-eight out of 64 participants (75\%) showed this pattern.

To summarize, these analyses indicate that people’s choices reflect an asymmetry between gains and losses. Of two proposals concerning the specific nature of such asymmetries, we found support for only one, namely Wu and Markle’s (2008) hypothesis that probability sensitivity is reduced in mixed as compared to single-domain gamble problems. Yechiam and Hochman’s (2013a) proposal of a higher choice sensitivity for gambles involving losses was not supported (in fact, we found the opposite pattern). Next, we turn to the question of how well the different models of the weighting function accounted for individual differences in choice sensitivity.
implementations of CPT summarized in Table 1 can account for people’s choices. For instance, even if there is support for Wu and Markle’s (2008) hypothesis of a lower probability sensitivity in mixed (as compared to single-domain) gambles, does an implementation of CPT allowing for this pattern (i.e., CPT_{gsm}) also perform well in terms of BIC?

**Model Comparison**

Figure 1 shows the median (across participants) BICs for each of the CPT implementations. As can be seen, the best-performing model is CPT_{dfixl}, which allows for gain-loss asymmetries in the elevation of the probability weighting function but sets $\lambda = 1$. Figure 2 shows the probability weighting function of CPT_{dfixl} based on the median best-fitting parameter values. The figure shows that this model represents a gain-loss asymmetry by having a more elevated weighting function for losses than for gains, $\delta^- = 1.69, \delta^+ = 0.63$. Like models implementing the utility account, CPT_{dfixl} gives more weight to losses than to gains, but does this via the decision weights resulting from the weighting function rather than via the value function. CPT_{dfixl} not only achieved the best performance in terms of the median BIC, but also the overwhelming majority of individual participants (54.7%) were best accounted for by this model.\(^3\)

Our results seem to challenge the approach taken in previous tests of prospect theory that have focused on specific and individual gamble problems. For instance, using problems specifically designed to test gain-loss separability, Wu and Markle (2008) found evidence for a superiority of a version of CPT that allowed for different probability sensitivity in single-domain than in mixed gamble problems. By contrast, in the data set used here, where the gamble problems were not constructed to test specific assumptions of CPT (instead many of the gambles had been randomly generated; see Glöckner & Pachur, 2012, for details), Wu and Markle’s modified version of CPT performed rather poorly (Figure 1). The results of our model comparison thus suggest that model developments based on focused tests may sometimes sacrifice a model’s

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\(^3\) The second-best model in terms of selection frequency was CPT_{nola}, which best accounted for 17.2% of the individual participants. Interestingly, the second-best performing model in terms of the median BIC, CPT_{l}, best accounted for only 4.7% of the participants.
ability to account for choices more generally for its ability to account for idiosyncratic cases.

What does the superiority of the version of CPT with a more elevated probability weighting for losses than for gains (Figure 2) mean psychologically? The cognitive underpinnings of probability weighting are still rather little understood. This has led some researchers (e.g., Brooks & Zank, 2005; Zank, 2010) to focus more on what can be called “behavioral gain-loss asymmetries”, that is, specific choice patterns that follow from gain-loss asymmetries on the value and/or probability weighting functions.

These open questions notwithstanding, our results suggest that if one’s goal is to predict how people will decide between risky alternatives, modeling gain-loss asymmetries in terms of differences in probability weighting rather than utility weighting promises to be a more successful approach. Our conclusions thus resonate well with Prelec’s (2000) assessment that “probability nonlinearity will eventually be recognized as a more important determinant of risk attitudes than money nonlinearity.” (p. 89)

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References


