Topological Similarity of Motor Coordination in Rhythmic Movements

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Abstract
Recognition of motion is vitally important to any animal. Vision research has proposed a number of algorithms applicable to action recognition. However, unlike successes in early visual perception, the past studies have not yet established the computational theory of action recognition. In the present study, we employ a dynamical systems approach and hypothesize that motions are encoded cognitively as a topological structure abstracted from physical particulars. We investigated whether a common topological nature could be found in a type of rhythmic movement. The topological nature of action dynamics showed a striking similarity, which could not have been identified with other analyses where physical properties were retained. The result suggests that the dynamical perspective serves as a theoretical basis in studying complex human movements.

Keywords: Actions Recognition; Motor Coordination; Dynamical Systems; Invariant Measures

Recognition of Actions
Recognition of motion is vitally important to any animal. Detection of another animal, whether predator or prey, or a conspecific, and subsequent detailed identification of the other and how it may behave is essential to taking any emergent actions (Johnson, Bolhuis, & Horn, 1985). Not surprisingly, our visual system is highly specialized to recognize others’ actions. How do we recognize bodily movements? Our main focus is that, despite much of the advances, we still miss a parsimonious explanation of “what is an action” or a computational theory of actions. The goal of the present study is, thus, to propose a computational-theory level description of actions which abstract identities beyond physical particulars. We briefly review psychophysical findings and theoretical works on action recognition.

The past experimental literature has explored the capacity of motion perception using point-light displays (Johansson, 1973) in which the point-lights attached to major joints are only visible in the dark background. The available information is point-wise kinematic motion of multiple body parts. Despite the limited information, people can recognize identity (Troje, Westhoff, & Lavrov, 2005), gender (Kozlowski & Cutting, 1977; Troje, 2002), emotions (Pollick et al., 2001; Atkinson; 2009; Hobson & Lee, 1999), dynamics such as the weight of a lifted object (Bingham, 1987) of actions from point-light displays. Accumulating empirical studies on action perception have suggested that velocity and its higher order derivatives in single or multiple body parts characterize actions: duration of action (Pollick et al., 2001), velocity (DeMeijer, 1989), acceleration (force or the second order time derivatives) (Chang & Troje, 2008; 2009), jerk or the third order time derivatives (Cook, Saygin, Swain, & Blakemore, 2009), and pairwise counter-phase oscillation (Chang & Troje, 2008; 2009).

Consistent with these empirical findings, most of the theoretical approach employs statistical regularities among motion profiles (Hidaka, 2012). According to a recent review (Troje, 2008), perception of biological motion involves multi-level processing on local and global motion properties. Feature processing consists of four layers from early (low-level) to late (high-level) processing: life detection, structure-from-motion, action recognition, and style recognition. The system detects an autonomous agent, and constructs body structure from its detailed analysis, then is followed by more detailed action analysis.

A couple of models have been proposed for structure-from-motion and action recognition (Giese & Poggio, 2003; Lange & Lappe; 2006). In the model of structure-from-motion and action recognition, the model identifies body structure, and subsequently recognizes actions from the pixel-based visualization of point-light displays. In Giese & Poggio (2003), the model was built based on neuro-physiological findings in the visual cortex, and was applied to recognition of action types and action direction in normal, masked, or scrambled point-light displays.

Despite the accumulated empirical evidence and theoretical works, its computational level account attributed to Marr (1982) – a description of function, i.e., set of input–output pairs in action recognition – is still missing. The two models above constructed algorithms which recognize a class of actions or properties of actions through processing the features of human bodily movements. However, in general, algorithmic models formalize specific procedures, but their meaning is not often readily apparent. First, a complex model typically loses transparency of mechanism as a cost of generality (For example, multi-layered physiologically-plausible model, Giese & Poggio, 2003). A drawback of complex models (using nonlinear filters or feature decomposition technique) is that the estimated parameters do not necessarily offer a clear interpretation on which attributes are informative in the recognition processes. Second, such a complex model often outperforms human recognition (Troje, 2002; Davis, & Geo, 2004; Pollick & Paterson, 2008) rather than explaining it. It is thus dubious whether those models can explain the action recognition of human beings.

We are interested in the computational level of action recognition rather than in the algorithmic level. We study actions, that is, how our multiple body parts are coordinated...
in performing particular tasks. Our human body consists of over two hundreds bones, numerous muscles, and billions of neurons in the central and peripheral nerve systems controlling them with feedback loops. Obviously, making a smooth action requires integrated control across all levels of these interactive systems. Given this complexity in motor control process, it is unrealistic to compute the inverse transformation (as supposed in early vision) from end-point visible actions to its intrinsic motor control patterns. Therefore, instead of such inverse computation, we hypothesize that the goal of action recognition is to compute “dynamical invariances” under smooth transformation. This hypothesis views motor control underlying human movements as a set of dynamical systems, that is, a sequence of interactions between elements involved in controlling movements such as body joints, muscles, neural systems, etc. Our hypothesis can be best understood in the context of the dynamical system perspective on the motor coordination (Shaw et al., 1996; Turvey, 1998; Smith & Thelen, 2003). The properties retained in dynamical systems for long term can be captured with invariant measures such as attractor dimension or Lyapnov exponent (Kantz & Schreiber, 1997).

We define a higher dimensional space, i.e., phase space, within which all possible combinations between elements involved in controlling movements can be found. An action is then defined as a trajectory on the space. Trajectories can be projected onto lower dimensional spaces, e.g., actual movements observable from outside. In our study, we collect motion data to reconstruct the dynamical systems by embedding the time series in a higher dimensional space. For graphical examples, Figure 1 illustrates attractors, or the state space which the system may take in the three theoretical dynamical systems, the Hennon map, Rossler system, and Lorenz system (Figure 1 (a-1), (a-2), (a-3)). A univariate time series (as imperfect observation of the system) is shown in Figure 1 (b) for each of these systems. Since the original systems live in two or more dimensions, these univariate time series do not have full information due to missing dimensions. Thus, we need to “reconstruct” the phase space instead of studying the degenerated patterns. By taking the time delay vector (e.g., \( x(t), x(t+\delta) \)), the topological nature of the phase space is reconstructed (Figure 1c). In Figure 1 (c-1)-(c-3), the time-delay embedding (a map from low to high dimensional space) successfully recovers similar topological structure shown in Figure 1 (a-1)-(a-3) only from the degenerated data Figure 1 (b-1),(b-3). Although the original phase space is unknown for empirical bodily movements, we expect the intrinsic topological nature can be reconstructed in the same way as the theoretical dynamical systems (Figure 1 (b-4) and (c-4)). See Kantz and Schreiber (1997) for a detailed description of these procedures for nonlinear time series analysis.

To study dynamical invariances, we investigated topological similarities of motor coordination. The rationale for the approach is found in observations such that one can mimic other’s behaviors no matter how different their individual appearances. Topology abstracts over physical particulars such as distance, speed, etc., to extract some dynamical invariances independent of these physical properties. Specifically, we examined the dynamical properties of rhythmic movements for two main reasons. First, rhythmic movements are not just a period but with fluctuating accents, and this is expected to show complexity to some extent neither too simple nor too complex. Second, actions which an actor can maintain continuously and produce a substantial amount of datasets are necessary for characterization of dynamical invariances.

![Figure 1: Phase space of (a-1) the Hennon map, (a-2) the Rossler system, (a-3) Lorentz System, and (a-4) the body model and attached markers (filled circles: analyzed, open circles: attached but not reported in this study). (b1-3) A univariate time series from the original phase space in (a1-3) (b-4) An x-axis phase of the Shaker 1 in the expert player in the 60-bpm trial (blue circles) with the estimated noise-reduced time series (black dots). (c1-4) The reconstructed phase space from the low dimensional observed time series in (b1-4).](image)

**Chacterizing Complex Rhythmic Actions**

The data was originally obtained in order to analyze the levels of expertise in the samba music plays (Yamamoto, Ishikawa, & Fujinami, 2006; Yamamoto & Fujinami, 2008). The dataset consists of five players, and each player performed basic samba shaking actions in five different tempos (60, 75, 90, 105, and 120 beats per minute, and each trial lasted 97.4 seconds on average) by being cued with a metronome. While playing, three dimensional motions of 18 markers, attached on body parts and musical instruments, were recorded at 86.1Hz of sampling rate (Figure 1a-4). As well as the original study, here we aim to find the relationship between dynamical properties among bodily actions. For simplicity, we limited ourselves to analyze a subset of the original datasets, 3190 samples (74.1 seconds long) of four markers attached on the right wrist, right elbow, and two sides of the musical instrument (shaker), having the right shoulder as a reference point (Figure 1a-4). These were the essential parts of the samba actions making sounds directly, and we
expected that dynamic coordination among them would be crucial to characterize the dynamical properties of the samba.

**Preprocess and phase space reconstruction**

In the analysis, after down-sampling the original data to 46.05 Hz, the first 250 samples (5.81 second long from the beginning of the recording) were excluded as initial setup of the actions, and 3250 samples (75.5 second long) of velocities were analyzed for each subject. In order to reduce measurement noise, for each movement of the markers, the local linear projective method was performed after phase space reconstruction of each time series on the 31 dimensional time delay space with 46 msec (i.e., \{ t, t + \Delta t, t + 2\Delta t, \ldots, t + 30\Delta t \} where \( \Delta t = 46 \) msec) (Takens, 1981). This technique is a nonparametric and unsupervised method which, in principle, reduces observation noise independent of the time series intrinsically generated from a nonlinear dynamical system. Figure 1 (b-4) shows the original data (open circles) and its noise-reduced data (filled circles) after applying the local linear projective method. Due to digitalization in the motion capture system, the original data only takes certain discrete values which may be potential sources of observational noise in the measurement. As the result of the noise reduction, we obtained the 31 dimensional phase space of 3220 points for each coordinate of three dimensional positions of each marker movement in each subject and trial. An example of the reconstructed phase space is shown in Figure 1 (c-4).

**Estimating symbolic dynamics**

For each estimated phase space (Figure 1c-4 and Figure 3), a symbolic dynamic is estimated by the symbolic false nearest neighbor method (Buhl & Kennel, 2005; See also Hidaka & Yu, 2010). In this algorithm, a symbolic state is assigned to each data point in the given phase space by minimizing the error in the one-to-one correspondence between spatial nearest neighborhood and symbolic nearest neighborhood. After convergence of the iterative minimization procedure, we obtained the series of binary symbolic states for the trajectory in the high dimensional phase space. Each symbolic series of length 5 (e.g., the subsequence “01100” as a state) is reported as a state in the present study, but we found the similar results consistently with the symbol length from 5 to 8. It means that the state transition in each phase space is analyzed by partitioning into 32 (i.e. 2^5) distinct states.

**Results**

In order to see the rhythmic properties as phase shifts in repeating actions, we analyzed the temporal profiles of the velocities in the right arm and wrist. Figure 2 shows the histogram of phase differences between body parts with the right shoulder as the reference point. Since the right elbow and wrist are the major body parts playing the shaker, their temporal structure was expected to reflect the rhythmic characteristics. However, not as expected, the five musicians showed quite different distributions in terms of phase shifts among body parts, even for the right wrist and elbow movement to play the shakers to the same tempo. The peaks found for the elbow and wrist are sharp for musicians A and D, compared with those found in the other musicians. As for A, the peaks of the elbow and wrist come to the same phase, but the peak is less visible for the wrist. As for D, the peaks of the elbow and wrist come later than that of the shoulder. For the other musicians, no obvious feature is found. The frequency uniformly varying over the phase angle shows large fluctuations in arm movements for each musician. The histograms revealed both within-musician fluctuations and individual differences rather than similarity among actions.

The results suggest that characterization of the “same” action (i.e., playing to the samba rhythm) on the levels of physical properties may lead quite different patterns across subjects. Needless to say, changing physical properties such as tempos also directly changes phase differences. The level of physical properties is not sufficient for characterizing actions even if the major parameter of the actions (i.e., tempo) is well controlled.

Next, we analyzed the properties of actions by looking into the dynamical systems underneath body movements. A basic technique to characterize dynamical properties from an empirical time series is phase space reconstruction. A phase space reconstructed by time-delay embedding is visualized as a three dimensional subspace projection (Figure 1c-4). The phase space is originally a set of velocity vectors of the four markers including two sides of the shakers, right wrist and elbow. The trajectory on the reconstructed phase space shows an attractor or the state space the system may take. The phase space is 124 dimensional space consisting of 31 time-delay copies of the four dimensional time series. First we analyzed the dimensionality of the attractors as one of invariance for the dynamical system. It is formally measured by correlation dimensions (Kantz & Schreiber, 1997), and we found the correlation dimensions varying from 1.8 to 2.4 across five musicians and five conditions. These results suggested the state space of the samba rhythm is rather restricted on a low dimensional space.

Since the dimensions of the attractors are lower than three, it allows us to visualize them in the three dimensional space without losing much information. Figure 3 shows the attractors estimated for all five musicians on the five conditions. Visual inspection of the samba attractors grasps the gist of commonalities among the attractors. Consistently across most of the attractors, they share a similar shape of trajectories – a twisted double circle (which may appear different due to a specific visual angle of each attractor). These similar “shapes” of trajectories indicate that the topological nature of the attractors is similar.

In order to quantify the similarity among attractors, we performed analysis based on symbolic dynamics (Buhl & Kennel, 2005). Symbolic dynamics offers a way to analyze a topological property of a dynamical system by constructing a homeomorphism (map preserving the topological nature) from the original space to a symbolic space. If two state spaces are homeomorphic (topologically identical), we find identity between their symbolic dynamics
as well. Importantly, it is easier to compute similarity between two symbolic spaces than that between two high-dimensional phase spaces. This mathematical property is applied in our data analysis. By estimating the symbolic dynamics for each of the samba attractors, we estimated the probability of state space transition on the estimated symbolic space (Figure 4). Each of the top left two-by-two panels shows a probabilistic distribution (stationary distribution) of the symbolic states for each musician and condition. Each symbolic state is defined by a binary symbol series of length 5 (e.g., “01100”), and the distribution over its 32 possible states is shown in the figure.

In Figure 4, we show the analysis concerning a pair of musicians and conditions as representative results. Each distribution of symbolic states reflects the topological nature of the underlying dynamical systems – transitivity or connectivity of states in the system. As shown in Figure 4, the distributions of state spaces are quite similar across subjects (R=0.784 and R=0.962) shown in the panels of Figure 3 and across conditions (R=0.959 and R=0.803) shown in the panels in Figure 2. Across all the pairs of five subjects and five conditions, the average correlation of distribution is 0.831. This means that the topological nature of the complex actions as dynamical systems were quite robust across different musicians and different playing tempos.

As a baseline comparison, we performed cross correlation analysis of the physical movements of the shaker (the major axis of the 3D motions with the most variance). By taking a time lag maximizing correlation, the average cross correlation was 0.453 across musicians and trials (0.462 across musicians (the same tempo) and 0.478 across tempos (the same musicians)). These results showed the temporal correlation of the shaker movements were not as high as the correlation of the symbolic state space (0.831) even with the time lag optimally adjusted and the playing tempo being held constant. This result means that the similarity of movements as physical properties cannot explain the topological similarities shown above.

![Figure 2: The distribution of phase shift (radian) of right elbow (gray) and right wrist (white).](image)

![Figure 3: The reconstructed phase space embedding in three dimensional time delay space in Musician A-E playing at tempo 60, 75, 90, 105, and 120 BPM. The velocity of the trajectory in the three dimensional space is shown as RGB color code for its visibility.](image)

![Figure 4: Stationary distributions of states in each of symbolic dynamics (four top left panels) and their correlations between musicians and conditions (four right side and bottom panels).](image)

The result -- higher similarity between topological properties of the state space -- is quite surprising with consideration to the individual differences on the physical level characteristics (Figure 2) and low cross correlations in the physical movements. The results suggest that the topological properties of the attractors were quite similar across different musicians and tempos, while their physical realizations of the actions differed from person to person.

**Discussion**

One of the challenges to the theory of action recognition is formalizing the possible attributes of characteristic actions. In the present study, we hypothesize that an intrinsic topological nature of actions as dynamical systems...
characterizes a similarity between actions. This is meant to describe actions on the basis of invariances under nonlinear transformations, rather than the specific features (coordinate systems) the actions have. In other words, this is to abstract the actions from their physical properties. In order to test the hypothesis, we investigated the samba playing action, which is repetitive rhythmic movement. The samba rhythm fluctuates within a certain range, showing a complex accent pattern, even if an auditory cue is given to keep the tempo constant (Figure 2). No common property was found in movements across musicians in physical movements of the right arm.

Subsequently, we analyzed the same data from a perspective of actions as dynamical systems. In the analysis, we define a higher dimensional space in which the action is mapped as a trajectory. We analyzed time-delay vectors, which embed the time series of movements, i.e., lower dimensional data, into higher dimensional space.

The analyses revealed topological similarities in the reconstructed phase space among the musicians and among the different playing conditions. The analyses using the symbolic dynamics quantified the similarities in terms of their topological structures. In sum, these results supported our hypothesis that human actions can be characterized on the basis of invariances as dynamical systems. This invariant nature of the dynamical property can serve as a possible basis for our perception of actions, and offers an explanation of why we perceive them as “the same actions.” Interestingly, the patterns revealed in the current analysis (Figure 3) are not just abstract-level depiction, but they also correspond with the introspective view of the samba rhythm (Figure 5: obtained from the most experienced musician A in the post-experiment interview). His drawing represents a general periodic motion with accents at a particular part of the trajectory. The geometric shape of the trajectory closely corresponds with the reconstructed phase space (Figure 3).

Finally, we briefly mention the implications to the two relevant research fields. One is the imitation of bodily movements. It is necessary to map from the body of the self to the body of the other in order to imitate the other’s actions (Breazeal & Scassellati, 2002). One of the traditional approaches to this problem is to compute inverse kinematics (estimating the motor control parameters from the perceived actions) upon which a number of robotics applications have been based (for example, Wolpert, Doya, & Kawato, 2003). This approach, however, does not offer a sufficient explanation for neonatal imitations (Melzoff & Moore, 1977) and how actions are identified, because neonate do not have opportunity to learn the cross-modal identity of the action (i.e., visual patterns of the other’s action and motor control of the self). In turn, topological similarity of actions revealed in the present study may potentially offer a cross-body identity of actions regardless of their differences in physical particulars.

The present study proposes the dynamical perspective of actions in which it is essential to characterize topological similarities of actions as attractors. It is viewed as a paradigm shift from cognition as inverse computation for an ill-posed problem to the computation of invariances under smooth transformations.

Figure 5: A drawing by the expert in his introspective explanation of the samba rhythm.

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