Abstract

The present study examines the decisions made by reasoners when they are asked to revise their beliefs in the face of new, counterfactual information. Participants indicated the Scope (the degree of set inclusion) of semantic generalizations about real categories in a Pretest. In subsequent experiments, these Scope values were used to predict the willingness of participants to retain statements in their existing knowledge sets. When those sets were logically compatible with a Modus Tollens (MT) structure, participants were more likely to retain the general statements, but not when the sets were logically compatible with a Modus Ponens (MP) structure. However, the MP retention rates increased when locatives were added to the generalizations. These findings are inconsistent with several prevailing proposals of belief revision but do support the concept of belief revision as following Possible Worlds logic.

Keywords: belief revision; counterfactual reasoning; scope.

Introduction

The process of belief revision is known by many terms (e.g., belief updating, belief change or belief dynamics). Simply put, belief revision involves the possibility of changing a previously held belief in light of new and assumed true information. At its very basic level, this involves updating knowledge and resolving inconsistencies within a pre-existing knowledge structure.

True belief revision must involve commitment to true beliefs. The earliest belief revision studies employed dictionary definitions without proper verification that those definitions had merit or that the students were committed to those definitions (Revlis & Hayes, 1972). Later studies examined belief revision with artificial categories or groups, of which a participant might have had no previous knowledge (e.g., Byrne & Walsh, 2002; Elio & Pelletier, 1997; Politzer & Carles, 2001; Revlin, Cate, & Rouss, 2001). Revlin, Calvillo, and Ballard (2005) specifically created a fantasy world with Lego figures and various arbitrary rules about knights and kings (see also Van Hoeck, Revlin, Dieussaert, & Schaeken, 2012). In each of these cases, real beliefs were not tested. It is difficult to assess the process of belief updating when the epistemic system is limited to arbitrary or unverified knowledge. The findings from such studies have supported conflicting theoretical treatments (e.g., Byrne & Walsh, 2002; Wolf, Rieger, & Knauff, 2012; Revlin et al., 2001; Revlis & Hayes, 1972).

The focus of the present study is to identify the basic cognitive processes in true belief revision while still employing an established paradigm.

A basic paradigm for studying natural belief revision has been borrowed from the philosophical treatment of belief-contravening problems (Rescher, 1964): It consists of a set of beliefs that are relevant to a counterfactual assumption, whose introduction requires a revision of the belief set. For example,

(1a) All whales are mammals
(1b) This creature is not a mammal
(1c) This creature is not a whale
(1d) Assume that this creature is a whale

A typical adult reasoner with the current knowledge of statements (1a-1c) would appreciate the inherent consistency of these statements. This collection of statements follows the logical form of Modus Tollens (MT; \( p \rightarrow q, \neg q, \therefore \neg p \)). However, if someone is faced with statement (1d), an inconsistency is introduced to the knowledge structure and the revision process requires the reestablishment of a consistent epistemic set that entails the retention of (1d). To accomplish this, the reasoner notices that statement (1d) is in direct contradiction to statement (1c), which can be easily eliminated. However, there is a larger issue. The remaining statements (1a & 1b) jointly create a contradiction with (1d). To resolve this inconsistency, a choice must now be made: Does the individual accept statement (1a), retaining the previously held belief that all whales are mammals, and that it cannot be true that there is an creature that is classified as a whale that is not a mammal? Or does the individual accept statement (1b), and claim that it could be true that there are creatures classified as whales that are not mammals? The revision process requires the elimination of one of the two statements. Either path is equally logical, although standard logic fails to encourage a preference, only indicating to the reasoner that there is an inconsistency (Chisholm, 1946).

While an individual can reject both statements, the goal is generally to retain the maximum number of statements that already exist within the epistemic set. A second logical form typically used in belief-contravening problems is that of Modus Ponens (MP; \( p \rightarrow q, p, \therefore q \)), illustrated in (2) below:

(2a) All whales are mammals
(2b) This creature is a whale
(2c) This creature is a mammal
(2d) Assume that this creature is not a mammal
The assumption (2d) introduces the same direct and indirect contradictions as (1) above, and the reasoners must revise the belief-set to resolve the inconsistencies that are created. Notice that the MP assumption (2d) undermines the credibility of the generality (2a) by changing the properties of the specific instance (2b) that is already within a category—thereby making the category incoherent. In contrast, the MT assumption (1d) adds a new member to a category (1b) with seemingly different group membership or properties, while not changing the credibility of the generality as in the case of the MP assumption.

Despite the fact that standard logic is unable to guide the selection made by reasoners, they have shown distinct patterns of resolution for each of the logical forms, which vary with the content of the studies. In some studies, reasoners show a distinct preference to retain the generalities in problems like (1) above (e.g., Revlin et al., 2005; Revlin, Calvillo, & Mautone, 2003). In other studies, preference has been shown for creating disabling conditions (or exceptions to the generality or rule), which allows them to be supported with caveats (Khemlani & Johnson-Laird, 2011). In some studies, no preference among reasoners has been shown, especially for problems like (2) above (Byrne & Walsh, 2002). We propose here that these differential findings may be a consequence of the degree of reality of the beliefs to be revised.

In addition to examining the role of real beliefs, the present study will focus on the importance of Scope in belief revision. The Scope of a quantified statement specifies the instances of the generality (or rule) that are subsumed within it across time and space. To demonstrate, consider the statements: all the coins in my pocket on VE Day are silver (Goodman, 1954) and all whales are mammals (Ryle, 1949). While these statements are both universally quantified generalities, the first is considered an accidental generality (it just so happens that all the coins in the pocket are silver), and the second is a scientific law (which spans space and time). A reasoner would prefer to retain the second statement because of its law-like quality; such statements are intended to act as inference tickets in new situations (Ryle, 1949). A reasoner should regard the Scope of the first statement about the silver coins to be quite small. In contrast, a reasoner should recognize that the second generality regarding mammalian whales has a large Scope (imagine all the whales that have existed in the past, present, and future and classify them into the superordinate category of mammals). The Scope of these relationships is generally an important proxy for knowledge preservation and credibility. It is possible that belief revision with artificial categories employed generalities with restricted Scope, which impacted revisions.

In the experiments in this study, Scope is either presented implicitly (with statements from the Pretest) or explicitly (the inclusion of numbers in the generality expression), and the goal is to determine whether Scope affects belief revision. To gain a sense of a statement’s Scope at a granular level, we asked participants to indicate the number of instances of a large category that possess a critical property—e.g., the number of whales that are mammals.

A pretest was used to assess participants’ commitment to Scope values of general statements that were derived from theories of semantic knowledge (e.g., Collins & Quillian, 1969; Quillian, 1968; Rips, 1989). In Experiment 1, these generalities were included in a basic belief revision paradigm where problems took the logical forms seen in example problems (1) and (2). In Experiment 2, locatives were added to the statements to constrain the law-like quality of the statements and to de-couple real world categories from the reasoning context. In Experiment 3, the Scope of the statements was explicitly manipulated into small and large proportions of a given set (Scope) to determine the effects of explicitly stated Scope on the reasoning process.

Pretest

In order to create test materials for the belief revision task (Experiments 1-3), a pretest was developed to measure the implicit Scope values of various general statements.

Ninety-one undergraduates volunteered to participate in this pretest condition for course credit. They viewed 24 universally quantified statements. There were four conditions of statements created by crossing two levels of Ontology (Definitionally true or Empirically true statements) by two levels of Relation (Class-Inclusion or Property-Assignment).

Students identified the Scope of each statement on an 8-point scale, where each point corresponded to a power of 10. The scale was anchored by “0” and “7”. For example, a Scope of “4” encompassed a Scope size from 1,000 to 10,000. The order of the statements was randomized in the booklet.

As anticipated, Scope values were greater for Definitional statements ($M = 6.26, SD = .90$, e.g., All trees are plants) than Empirical statements ($M = 5.65, SD = .82$, e.g., All professors are teachers). Class Inclusion statements ($M = 6.09, SD = .88$, e.g., All oranges are fruit) received greater Scope values than Property-Assignment statements ($M = 5.82, SD = .83$, e.g., All mammals have hair). These findings are in keeping with the importance of these variables for semantic verification (e.g., Collins & Quillian, 1969; Quillian, 1968; Rips, 1989).

Experiment 1

The goal for Experiment 1 was to examine belief revision in the context of statements that have verified believability and Scope. Using the Scope values recorded from the Pretest, we aimed to compare those values with retention rates to determine the role of Scope in a statement’s retention. We predicted that for both MT and MP problems, retention rates would increase as the Scope of the generality increased because Scope reflects the law-like aspect of the statements.

Method
Seventy-seven undergraduate students participated in this experiment for course credit. Two booklets of belief revision problems were created from the Pretest materials. One booklet contained MT belief revision problems and the second contained MP belief revision problems. Within each booklet, there were four types of problems (Problem Type) that result from the crossing of Ontology (Definitional or Empirical relations and Relation (Class-Inclusion or Property-Assignment). There were three exemplars for each type of problem, chosen randomly from the Pretest, creating booklets of 12 problems each.

The problems were randomly ordered in each booklet and the booklets were randomly assigned to participants. For each problem, participants were asked to accept the assumption as true, and discard (by crossing-out) the statements that contradicted the assumption.

Results and Discussion

Participants’ preference to retain the generality was scored for each problem and compared with chance (50%).

Table 1: Mean (SD) Retention Rates for MT and MP Revisions in Experiment 1

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>MT</th>
<th>SD</th>
<th>MP</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definitional</td>
<td>Class</td>
<td>91%</td>
<td>(0.21)**</td>
<td>56%</td>
</tr>
<tr>
<td></td>
<td>Property</td>
<td>93%</td>
<td>(0.18)**</td>
<td>67%</td>
</tr>
<tr>
<td>Empirical</td>
<td>Class</td>
<td>80%</td>
<td>(0.25)**</td>
<td>46%</td>
</tr>
<tr>
<td></td>
<td>Property</td>
<td>71%</td>
<td>(0.36)*</td>
<td>48%</td>
</tr>
</tbody>
</table>

Binomial analysis: *p < .05, **p < .01, ***p < .001.

Table 1 displays the rate of retention of generalities and shows that participants prefer to retain the generalities in all MT problems significantly more often than chance, but preference for generalities in MP problems was not reliably different from chance (MT: M = .84, SD = .25 and MP: M = .54, SD = .40; F(1, 66) = 18.44, p < .001). Analysis also revealed a significant difference in retention rates for Problem Type, F(3, 198) = 13.98, p < .001. There was no significant interaction between Problem Type and Logical Structure, with both MT and MP following similar trends, F(3, 198) = 1.28, p = .28.

Simple regression analyses were conducted to determine if retention rates changed in relation to the Scope of the generality (derived from the Pretest). Overall, Scope (β = .81, p = .001) significantly predicted overall retention rates (F(1, 10) = 19.37, p = .001, R² = .66). Specifically, for MT contradictions, retention of the generalities increased with Scope (β = .89, p < .001; F(1, 10) = 37.39, p < .001, R² = .80). However, for MP contradictions, Scope (β = -.55, p = .06) was a negative predictor of commitment to generalities, F(1, 10) = 4.40, p = .06, R² = .31.

When reasoners seek to revise true beliefs in order to return consistency to a set of statements, they show a stronger commitment to the generalities when the logical structure was expressed as an MT argument than as an MP argument even though the generalities are identical in the two conditions.

The Scope of the generality correlates positively and strongly with the tendency of reasoners to retain them in a MT argument structure, but Scope was negatively related to retention of generalities in MP arguments.

Experiment 2

In this experiment, we once again assess the importance of Scope for belief revision. Here we try to constrain the Scope of the generalities by including a location in each epistemic set that implicitly constrains the generalities’ Scope in space and time. When a locative is introduced, does it undermine the impression that the generalities are true across space and time? For example, consider “All snakes slither.” This statement entails a large Scope (shown in Experiment 1). However, if a special desert is referred to (e.g., Rich lives in a desert where all the snakes slither), it invites the question whether there could be something peculiar about the location or why would it be introduced? Here we assess whether a specific location limits the Scope of the generalization and therefore the pattern of belief revision. Alternatively, perhaps the cognitive processes employed to uncouple the generalization from the location would result in enhancing the reasoner’s commitment to the generality.

Method

Seventy-eight undergraduate students participated in this experiment for course credit. Participants solved the same belief revision problems used in Experiment 1, but those in Experiment 2 introduced novel locatives for each problem. Participants were either given MT contradictions or MP contradictions of the same problem set to solve (between-subjects). Participants solved 12 problems in total.

Results and Discussion

Table 2 shows the retention of generalities for Logic Structure and Problem Type. It reveals that participants preferred to revise beliefs by retaining the general statements significantly more often than would be expected by chance (50%) for each condition. As the table reveals, reasoners show a preference in all conditions to revise beliefs by retaining generalities. Logical Structure was important to revisions: participants who solved MT contradictions (M = .88, SD = .33) were more likely to retain the generality of the problem than those who solved MP contradictions (M = .71, SD = .34; F(1, 63) = 7.69, p = .007). The Ontology of the statements’ relations was also critical, with participants retaining the Definitional generalities (M = .82, SD = .25) more often than Empirical ones (M = .77, SD = .27; F(1, 63) = 3.62, p = .06). There was no effect of Relation (Class vs. Property) and no interaction among the variables in this study.

Scope was not found to be a significant predictor of retention rates for either MT contradictions (β = .27, p =
.39) or MP contradictions (β = .21, p = .51). In fact, the trend line seen in Experiment 1 for MP problems reverses direction (from negative to positive). This shows that providing a locative altered the importance of Scope.

Table 2: Mean (SD) Retention Rates for MT and MP Revisions in Experiment 2

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>MT</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M (SD)</td>
<td>M (SD)</td>
</tr>
<tr>
<td>Definitional</td>
<td>90% (.35)***</td>
<td>74% (.36)**</td>
</tr>
<tr>
<td>Empirical</td>
<td>85% (.36)***</td>
<td>69% (.38)*</td>
</tr>
</tbody>
</table>

Binomial analysis: *p < .05, **p < .01, ***p < .001.

Overall, the presence of a locative still allows an effect of Logic: participants tended to retain the generality for MT problems more often than those participants evaluating MP problems. However, comparing across experiments, the retention of MP generalities increased at a much higher rate (t(64) = -2.19, p = .03) than those observed in Experiment 1, while no change was observed in the retention rate of generalities in the MT structures.

Experiment 3

The aim of Experiment 3 was to examine belief revisioning in artificial environments (e.g., the locatives of Experiment 2), but with real categories, whose Scope has been modified. Scope was expressed as either a small proportion or a large proportion of the total members of the reasoning categories (e.g., Kelly has a hive where 5 bees out of 104 insects have wings vs. Logan has a honeycomb where 91 bees out of 104 insects have wings). In Experiments 1 and 2, Scope was implied. Here, it is explicitly stated. We anticipated that explicitly stating the Scope of the generalities would—along with the locatives—decouple the categorical expressions from their normative senses and therefore make the belief revision context more artificial.

Method

Fifty-four undergraduate students participated in this experiment for course credit. In addition to the variables present in the task for Experiment 2 (Logic, Ontology, and Relation), a new between-subjects variable was added to the problem set: Scope proportion (Small or Large). There were four total conditions: MT Large, MT Small, MP Large, and MP Small. Participants solved 12 belief revision problems where Scope information was given either in a Small (5%) or Large (87%) Proportion. The instructions and procedure for this task were the same as Experiments 1 and 2.

Results and Discussion

Table 3 shows that participants prefer to revise their beliefs by retaining the generality (Binomials, p < .05). Overall, reasoners who solved MT contradictions (M = .89, SD = .29) were slightly more likely to accept the generality of the problem than those who solved MP contradictions (M = .77, SD = .29; F(1, 50) = 3.99, p = .05). Compared with previous experiments, the retention rates of generalities in MT contradictions were at ceiling and those of MP contradictions were higher than previous. Such increased retention rates overshadowed any effect of Scope, which was not a significant predictor for MT or MP problems in either Small or Large Proportion conditions.

Table 3: Mean (SD) Retention Rates for MT and MP Revisions in Experiment 3 for Small and Large Scope Conditions

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>MT</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M (SD)</td>
<td>M (SD)</td>
</tr>
<tr>
<td>Small</td>
<td>89% (.42)***</td>
<td>77% (.40)**</td>
</tr>
<tr>
<td>Empirical</td>
<td>85% (.49)**</td>
<td>70% (.48)</td>
</tr>
<tr>
<td>Large</td>
<td>95% (.42)***</td>
<td>87% (.40)***</td>
</tr>
<tr>
<td>Empirical</td>
<td>86% (.49)**</td>
<td>74% (.48)*</td>
</tr>
</tbody>
</table>

Binomial analysis: *p < .05, **p < .01, ***p < .001.

The locatives appear to influence the decision-making by de-coupling the artificial context from the implicit Scope of the categories, thus allowing increased retention of generalities.

General Discussion

Revising our beliefs when we are confronted with conflicting information is ubiquitous. Yet, the cognitive processes underlying this kind of reasoning are poorly understood because the prevailing research has not studied belief revision with consistent content. Some tasks have used artificial content, with no relational structure among the beliefs. In others, the artificial beliefs have been part of simple assertions or immersed in stories. In cases where presumed beliefs have been used, they are often not verified. The present study created belief revision conditions where the meaning of the statements and the degree to which they could be interpreted as scientific laws—their Scope—have been independently verified along with reasoners’ commitment to them.

By controlling the Scope of statements, we were able to identify the importance of the logical structure in which the belief revisions are contained. In an MT structure, reasoners prefer to revise their beliefs by retaining the most law-like generalities and by eliminating the particular statements that are inconsistent with the generalities. For the MT structure, the Scope of the generalization predicts the tendency to retain these true statements: the greater the Scope, the more likely will the generalization be retained. In contrast, when the epistemic structure is cast as an MP argument, reasoners do not show a preference for generalities or facts nor does Scope play an appreciable role.

Belief on its own is not critical to the belief revision process. We know this because the MT problems employed the same generalities as the MP problems, yet the former were retained significantly more often than the latter. These
findings show that critical to belief revision are the dual factors of (a) a statement’s Scope and (b) the argument’s structure in which the statement is immersed. Neither factor alone is sufficient to account for how people revise a set of beliefs. This leaves the question of why these two factors should dictate the belief revision process.

Pursuant to David Lewis’ theory of Possible Worlds (Lewis, 1973, 1986), we propose that when revising their epistemic system, reasoners imagine an organized possible world, closest to the current one. To accomplish this, they rank-order the beliefs in terms of degrees of necessity, with the law-like propositions given the highest ranking. In MT, the general statement tends to be the one with the greatest Scope and is the starting point in the revision process with the deletion of any statement inconsistent with it. In contrast, in MP, the assumption statement challenges the modal status of the generality, diminishes its ranking, and rending all statements equivalent (Rescher, 1963, 2007). As a result, no preference for retaining any statement is revealed. In this case, the generalizations in MP problems do not possess the same commitment post- assumption as they do pre- assumption. Hence, Scope is less predictive of the decisions to retain statements in these problems. However, when the context is rendered artificial (as with a locative) it enhances sensitivity to the implicit modal status of the generalization even in the MT logical structures.

The relation between believability of statements and the logic of the belief revision context can be explained by the Conditional Probability Hypothesis (e.g., Evans, Handley, & Over, 2003; Wolf et al., 2012). It states that for MP problems, the probability of the truth of the generality is zero in the face of the counterfactual assumption. To understand this, recall MP example (2) above:

(2a) All whales are mammals
(2b) This creature is a whale
(2c) This creature is a mammal
(2d) Assume that this creature is not a mammal

The conditional probability of the generality is stated as \( P(q|p) = \) probability of “mammal” given the rule stated conditionally as “if whale, then mammal”. The assumption statement (2d) states that the probability of being a mammal is zero. Therefore, \( P(q|p) = 0 \); therefore the probability that the rule is true is also zero. This leads to the expectation that there will be no discernible preference for retaining either the generality or the particular statement in MP. In contrast, consider the case of the MT argument repeated below:

(1a) All whales are mammals
(1b) This creature is not a mammal
(1c) This creature is not a whale
(1d) Assume that this creature is a whale

In the case where the \( a \) priori belief in the generality is greater than zero, the assumption does not alter that. Therefore, the preference for the generalities in this MT structure will typically be greater than what is found for MP. This will be true even though the general statements are syntactically identical in the two logical structures. In brief, the conditional probability hypothesis is able to account for the typical finding that belief revision varies with logical structure, all things being equal.

However, the contrasting preferences shown in Experiments 1 and 2 are not readily explained by the conditional probability hypothesis. The retention of the generalities in Experiment 2 is reliably greater than those in Experiment 1 and this is especially the case for MP. The problems differ in the presence of a locative in Experiment 2, which is intended to reduce the law-like properties of the generality by reminding the reasoner that the truth of these statements may be limited in space and time. These locatives should also reduce the \( a \) posteriori probability of the inclusive category (e.g., “mammal”) and reduce the retention of generalities especially in MP. The procedure produced the opposite result. So, while the conditional probability hypothesis has much to recommend it as an account for belief revision, more work needs to be done to understand the cognitive processes contributing to the retention of beliefs.

A second account of belief revision has focused on Disabling Conditions. The claim has been made that in the face of the counterfactual assumption, reasoners construct explanations for the inconsistencies (Khemlani & Johnson-Laird, 2011). These explanations focus on the generalizations because they contain many component elements and reasoners imagine that one of these elements has been disabled, thereby allowing for an inconsistency. The degree to which such disabling conditions are contrived is indirectly related to the strength of belief in the generality. This approach makes the following predictions: (a) generalities will be rejected in order to retain consistency in the epistemic set; (b) since rejection of the generalities are based on believability, the disabling conditions (and therefore rejection of the generality) will be equivalent across logical structure.

These predictions are not consistent with the present findings. Overall, generalities are retained more often than would be expected by chance. Generalities whose Scope is artificially low (Experiment 3) should show the effect of disabling conditions more so than when the Scope is artificially high, yet no difference in retention is shown for these types of statements. While the presence of potential disabling conditions may play a role in some aspects of belief-revision, it is clearly not the underlying mechanism employed for confronting the counterfactual assumptions.

**Conclusion**

We are obliged to revise our system of beliefs when we accept a new piece of information that introduces an inconsistency into our knowledge structure. Here we are faced with the task of retaining some old information and rejecting others. The present study examined how people perform this task when dealing with real beliefs and facts. Scope and Logical Structure were jointly important predictors of whether students would retain a statement when required to revise their epistemic sets. When the statements fit within an MT structure, reasoners organized
their revisions around the law-like generalizations. In contrast, when the statements fit within an MP structure, the participants did not show a preference in how they organized their beliefs. This suggests that the importance of belief strength is influenced by the structure in which they are immersed.

These findings lend empirical support to the philosopher David Lewis’ view that belief revision is characterized by Possible Worlds logic in which reasoners structure their revisions by organizing their epistemic systems so as to give priority to the most law-like statements.

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References


