Constraints, Inferences, and the Shortest Path: Which paths do we prefer?

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Abstract

How do we reason about incomplete spatio-temporal descriptions? How might a map influence formerly constructed preferred mental models? Little research so far focused on a combination of two central fields important for successful route planning: the way humans deal with constraint based reasoning (especially with some sort of spatio-temporal constraints) and the way in which humans plan with a given map (especially with problems inspired by typical Traveling Salesman Problems). This, however, becomes even more interesting in cases in which the spatio-temporal constraints allow for several solutions. Do the predictions of the preferred mental model theory still hold true in such situations? This article investigates the influence of maps on the generation of preferred models. The goal is to bring together the theory of (preferred) mental models and route planning.

Keywords: Spatial reasoning; preference effects

Introduction

In everyday life we often reason with incomplete information or have to take constraints into account during reasoning. Cognitive processes involved in such reasoning about spatial relations and the construction of according mental models have recently been the subjects of interest in studies about spatial relations (Knauff, Rauh, Schlieder, & Strube, 1998; Rauh et al., 2005). However, the question of how external representations of space such as maps, or map-relevant knowledge influences and interacts with reasoning processes is widely unknown. The research communities concerned with how people use maps to solve spatial or navigational problems and how people solve reasoning problems are mostly distinct. There are, however, many situations in which people reason with maps or with map-like knowledge. In this paper we present and investigate two classes of problems.

Path planning from maps. Imagine planning a sightseeing trip through the downtown area of an unfamiliar city: you do have a map and you want to visit multiple sites of interest. Of course, you are interested in minimizing the distance you have to traverse along your tour. Problems of this kind are typically referred to as Traveling Salesperson Problems (TSP): A salesman has to visit a number of cities and start from a specific location to which he will also return after visiting each city. The traveling salesman will aim for the shortest possible route and avoid any detours (Wiener & Tenbrink, 2008). Formally, TSP-Problems are NP-complete (Garey & Johnson, 1979).

Human performance and the cognitive strategies employed when solving TSPs have been investigated in real environments involving movement through space (e.g., Gärling & Gärling, 1988) as well as in more abstract visual or map-like versions of the TSP in which a number of dots are displayed on a computer screen which have to be connected such that the resulting tour is as short as possible (e.g., MacGregor & Ormerod, 1996). When planning actual site seeing trips, however, we often face additional constraints besides minimizing distances: some sites of interest may close before others and therefore have to be visited earlier. Or, you may want to be at a specific site at a particular time, for example, to have lunch. In addition, you are still striving to minimize path length. Similar challenges arise when planning shopping trips during which multiple stores have to be visited. Here, we also often face additional constraints besides minimizing distances: Frozen food or ice-cream, for example, is best be bought towards the end of the shopping tour to avoid defrosting before returning home. Moreover, in order to minimize the effort of carrying purchased goods, heavy items should be bought towards the end of the trip. Again, path length should be minimized. All these factors impose constraints on the path-planning problem and have to be taken into account when planning a trip. Below is an example of combined spatial optimization and reasoning problem:

(1) Buy bread before ice-cream.
Buy eggs after ice-cream
Buy a gallon of water after eggs.
Buy a chair after a gallon of water.

Problems (1) belongs to a class of problems that are referred to as *determinate problems*, as they allow only for a single solution:

bread ice-cream eggs water chair

Problem (2) belongs to a class of problems that are referred to as *indeterminate problems*, as they allow for multiple – three – solutions.

(2) Buy bread before ice-cream.
Buy eggs after ice-cream.
Buy a gallon of water after ice-cream.
Buy a chair after a gallon of water

Which is consistent with the following three models:

bread ice-cream eggs water chair
bread ice-cream water eggs chair
bread ice-cream water chair eggs

The key idea of the *mental model theory* is that reasoners translate these constraints into a mental model – an abstraction or analogical reflection – of the state of affairs and use this representation to solve the reasoning problem. An important finding is that when faced with indeterminate problems featuring multiple solutions, humans tend to construct only one initial model – the so-called *preferred mental model* (Rauh et al., 2005; Ragni, Fangmeier, Webber, & Knauff, 2007), which is easier to maintain in working memory than any other mental model (Ragni et al., 2007; Knauff, 2006). Preferred mental models have been initially identified for Allen's interval calculus (Knauff, Rauh, & Schlieder, 1995), a more detailed introduction of preferred mental models is given in the next section. What happens when reasoning about a problem – as the one described above – when the shortest path does not correspond to the preferred mental model? Is any influence measurable? Although this question is of high ecological validity, to the authors' knowledge, it has not yet been approached. In this paper we will present a first experiment to analyze from the perspective of a mental model theorist whether – and if so, how – preferred mental models can be "overriden" by external stimuli.

Background

The theory of preferred mental models

A central question in the context of incomplete information is: How are indeterminate problems such as Problem (2) processed? Are there preferred interpretations?
The mental model theory (MMT), introduced by Johnson-Laird and Byrne (1991), suggests that people draw conclusions by constructing and inspecting a spatial array that represents the state of affairs described in the premises. It is a three-stage process consisting of a comprehension, description, and validation phase. In the comprehension phase, reasoners construct a mental model that reflects the information from the premises. If new information is encountered during the reading of the premises it is immediately used in the construction of the model. During the description phase, this model is inspected to find new information that is not explicitly given in the premises. Finally, in the validation phase alternative models are searched that refute this putative conclusion. However, some questions remain open with regards to how people deal with multi-model problems. For example, which model is constructed first, and does this model construction adhere to certain principles? And, why do reasoners neglect some models? None of these questions are answered by the classical mental model theory. In contrast the preferred mental model theory (PMMT) has been developed to explain why humans in general tend to construct a preferred mental model (PMM). The PMM is the starting point for deriving a putative conclusion. In the model variation phase the participants tend to make local and continuous transformations starting from the PMM to search counter-examples (Rauh et al., 2005).
Several predictions of the PMMT about insertion principles as well as transformation strategies in spatial relational reasoning can be shown (Ragni et al., 2007). Assume we have two premises of the form

(1) A is to the left of B and
(2) A is to the left of C.

Humans tend to process these premises sequentially, i.e. first a model A B is generated and then object C is inserted into the model. There are two possibilities where C can be inserted, in-between A and B (first-fit principle) and to the right of B (first-free-fit principle). The latter principle has been empirically confirmed in small-scale descriptions (e.g., Ragni et al., 2007; Jahn et al., 2005). An interesting aspect, however, is how this might influence reasoning if a map is given?

Path planning and Distance Optimzation. Path planning and optimization with maps has primarily been investigated by means of visual versions of the TSP (e.g., Graham, Joshi, & Pizlo, 2000; Vickers, Butavicius, Lee, & Medvedev, 2001). In these experiment, participants are presented with a number of target locations on a computer screen – usually presented as identical black dots on a white background – and are asked to connect these locations with straight lines such that the resulting path is as short as possible. Results from these studies demonstrate that humans reach very good performance levels even with as many as a few dozen target locations. The strategies and heuristics applied are a matter of ongoing debate. The convex hull has been suggested to be part of the problem solving strategy (MacGregor & Ormerod, 1996), the crossing avoidance hypothesis states that participants avoid crossing tours, as they know that crossings lead to sub-optimal solutions (Van Rooij, Stege, & Schactman, 2003), and the hierarchical nearest neighbor strategy assumes that in a first step clusters of several neighboring dots are established, which are then sequentially linked into a tour, using the nearest neighbor algorithm (Vickers, Bovet, Lee, & Hughes, 2003).

Only few studies investigating TSPs with maps used richer environments in which different target locations could be visually distinguished requiring some form of memory. In a recent study, Tenbrink and Wiener (2009) presented participants with maps depicting a regular 5x5 grid of locations each of which could be identified by a unique symbol. Participants were given so-called shopping lists depicting the symbols of a start location and four to nine target locations. Their task was to identify the locations in the grid and then mark the shortest possible round trip from the start that visits all target locations in the map. By analyzing participants' planning performance, their chosen paths, as well as retrospective linguistic representations, a number of cognitive strategies applied when solving the TSPs could be identified. Most importantly, participants flexibly employed and connected a repertory of multi-faceted strategies allowing them to simplify and structure the problem space across subtasks involved in solving the TSPs (for a navigational version of this paradigm, see Wiener, Ehbauer, & Mallot, 2009).

As mentioned before, path planning in every-day life often requires taking into account additional constraints besides minimizing distances. Hayes- Roth and Hayes-Roth (1988) presented one of the view studies investigating complex planning from maps with additional constraints (but see also the related Plan-A-Day paradigm, Nellen & Funke, 2002).

Participants were given a map of a town depicting multiple shops and other locations along with a list of errands. These errands included buying vegetables at the grocery, buying a toy for a dog at the pet store (both purely spatial constraints), but also picking up a car at a certain time in a certain location (spatio-temporal constraint). Moreover, more errands were specified than the subject could possible accomplish in the time available, which required him/her to sort out (less important) errands to formulate a realistic plan. Hayes-Roth & Hayes-Roth developed a general model of complex planning, assuming that the planning process comprises many distinct *specialists* contributing decisions to a tentative plan that is refined incrementally.

Experiment – Reasoning, Route Planning, and Maps

In this experiment we investigated the connection between the construction of (preferred) mental models from a set of premises and the subsequent task of planning a trip consistent with the premises. In order to do so, participants were presented with determinate and indeterminate reasoning problems describing spatio-temporal relations between sets of destinations. After processing the premises and (possibly) constructing a (preferred) mental model, they were asked to draw a round trip into a map visiting the destinations in an order that is consistent with the premises. If the planning task in fact interfered with the constructed mental model, we expected performance differences depending on whether or not the round trips defined by the premises were optimal or clearly sub-optimal with respect to path length.

Participants.

Nineteen students from the University of Freiburg took part in this experiment (9 females, *M* = 23.3/22.1*,* *SD* = 2.2/2.1). They were paid for their participation or received course credits.

Materials.

To investigate the impact of map like presentation of target locations on reasoning performance and the selection of preferred mental models, we generated four types of reasoning problems (see Fig. 1).

1. **Optimal determinate problem (D-optimal)**: The correct solution to these reasoning problems always matches the shortest possible – optimal – route to visit all target destinations.

2. **Suboptimal determinate problem (D-sub-optimal)**: The correct solutions to these reasoning problems were clearly suboptimal with respect to their length.

3. **Preferred optimal indeterminate problems (IP-optimal)**: The preferred mental model to these reasoning problems matched the shortest possible – optimal – route. Two alternative correct solutions existed that were not identical with the preferred mental model and that were sub- optimal with respect to their length.

4. **Preferred suboptimal indeterminate problems (IP-suboptimal)**: The preferred mental models to these reasoning problems were clearly suboptimal with respect to their length. Two alternative correct solutions existed, one of which was optimal with respect to metric length.

Methods.

Each participant was presented with 16 reasoning problems, four of each type described above. To control for the influence of the specific configuration of start and target places, we used four different configurations and balanced the types of reasoning problems across the configurations. Each reasoning problem was presented on three pages: The first page contained the first two premises; the second page contained the third and fourth premises, and the third page contained a regular 5 × 5 grid in which the 5 positions mentioned in the premises were marked (see Figure 1). Participants were instructed to read premises 1 and 2, to turn the page over, read premises 3 and 4, turn the page over, and to connect the positions in the layout in order to mark a round trip that was consistent with the premises. They were instructed not to scroll back after having turned a page or to take any notes.

Hypotheses & Predictions.

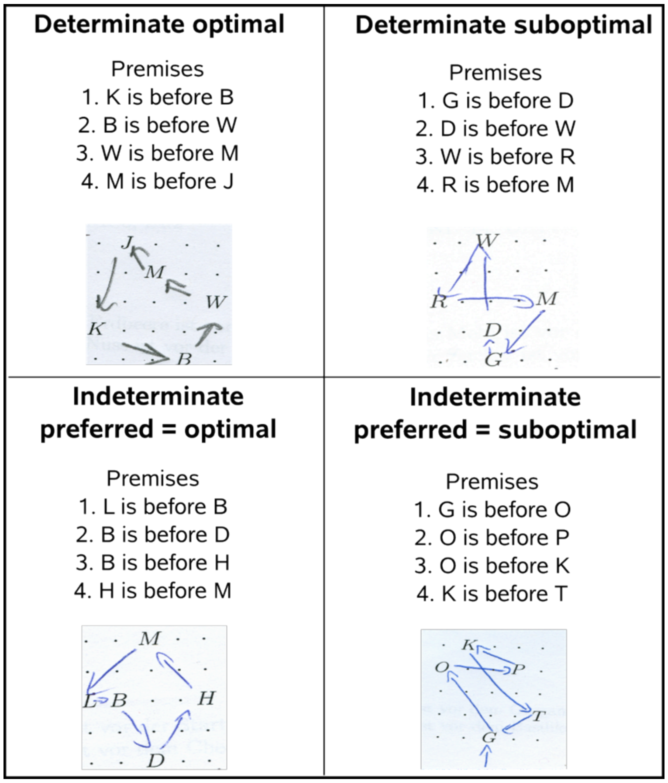
Given the specific procedures of the experiment, two competing hypotheses are conceivable: **First**, the external representation and the task of sketching the corresponding route do not influence the reasoning process. This is based on the assumption that the mental model is generated while reading and processing the premises. Hence, the external representation that is provided only after the last premises was processed does not influence the mental model. Participants would then simply sketch the tour that corresponds to their mental model. In case of determinate problems this would lead to identical performance (with respect to error rate) between the types of reasoning problems (D-optimal/D-suboptimal). In case of indeterminate reasoning problems, we expect that participants will select the preferred mental model, regardless of whether or not the according path was optimal or suboptimal (IP-optimal/IP-suboptimal). **Second**, the external representation influences the mental model, as the task of sketching a round trip for the shopping route implicitly requires choosing a short solution. In this case, we expect interferences between finding the correct solution to the reasoning problem and planning the shortest path. Such an interference would have a selective impact on performance for determinate reasoning problems of type *D-suboptimal*, for which the shortest (optimal) path and the correct solution to the reasoning problem were different, but not for determinate reasoning problems of type *D-optimal*, for which the optimal path and the correct solution to the reasoning problem were identical. The predictions for indeterminate problems are not as straight forward, as each indeterminate problem features three different solutions.

Results.

Three out of the 19 participants were removed from the final data as their performance on finding the correct solution for determinate problems was clearly below 50% (12,5%, 12.5%, 37.5%). In addition, thirteen trials were removed from the final data set, as these solutions featured branching points – participants had drawn two arrows from one location.

The different spatial configurations had no influence on participants' performance (F(3, 46.35)=.27, p=.85). For the remaining analyses we therefore pooled the four different configurations. On average, participants found a correct solution to 89.1% of the reasoning problems. A 2x2 ANOVA with the factors of *type of reasoning problem* (determinate, indeterminate) and *solution* (optimal, suboptimal) was carried out. We did not observe a main effects for type of reasoning problem [F(1,17.99) = 0.12, p = .91] or for solution [F(1,18.28) = .06, p = .81]. However, the interaction type of reasoning problem x solution was significant [F(1,16.72) = 8.96, p < .01].

To further investigate the nature of this interaction, we performed post hoc *t* tests revealing that performance for determinate problems of type *D-optimal* was better than for determinate problems of type *D-suboptimal* (93.2% vs. 82.8%; *t*-test: t(15)=2.24; p=.04, see Figure 2). For indeterminate reasoning problems, the pattern was different: surprisingly, participants performance was better for problems of type *IP-suboptimal* than for those of type *IP-optimal* (97.8% vs. 82.8%; *t*-test: t(14)=2.38; p=.03, see Figure 2).



Determinate optimal

Premises
1. K is before B
2. B is before W
3. W is before M
4. M is before J

Determinate suboptimal

Premises
1. G is before D
2. D is before W
3. W is before R
4. R is before M

Indeterminate preferred = optimal

Premises
1. L is before B
2. B is before D
3. B is before H
4. H is before M

Indeterminate preferred = suboptimal

Premises
1. G is before O
2. O is before P
3. O is before K
4. K is before T

Figure 1: The four different types of reasoning problems along with exemplary data by participants. All participants received the premises (in German) with full names, e.g. der Trevibrunnen vor dem Kolosseum (the Fountain of Trevi before the Colosseum) instead of initials.

For correct solutions to indeterminate problems, we evaluated whether or not participants chose the preferred mental model. In 87.8% of the cases, they did choose the preferred mental model (*t*-test against chance level [with three possible solutions, chance level was 33.33%]: t(15) = 15.79, p < .001).

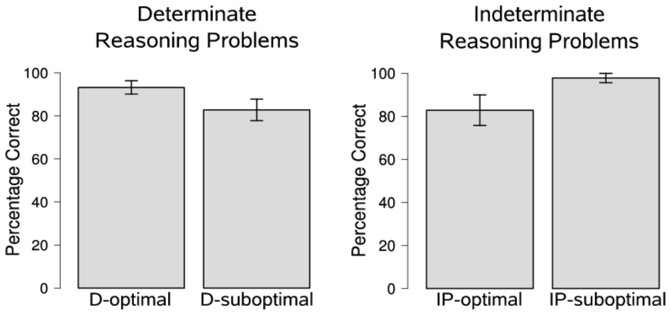


Figure 2: Results of the Experiment: **left:** the determinate problem description – allowing for one path solution only; **right:** the indeterminate problem description, allowing for three solutions.

Preference for the preferred mental model did not differ between types of indeterminate problems (IP-optimal: 87.2% versus IP-suboptimal: 85.0%: *t*-test: t(14)=0.29; p=.78).

Discussion.

The findings for the determinate problems to which only a single correct solution exists, revealed a clear difference in performance. Specifically, participants showed better performances for problems in which the correct solution to the reasoning problem was identical to the shortest possible path (D-optimal) as compared to reasoning problems in which the correct solution and the optimal path were different (D-suboptimal). This finding suggests interference between the reasoning process and the task of planning a round trip. In other words, the map influenced the reasoning process.

Contrary to classical deduction tasks, indeterminate reasoning problems do not appear to be more difficult than determinate ones. A possible explanation for the lack of a systematic difference in the current paradigm comes from the fact that the higher number of possible solutions in indeterminate problems in the specific task allows for a higher error tolerance (in all cases the objects 3, 4, and 5 in the premises).

Some of the participants had drawn routes with branching points, i.e., they had drawn two arrows from one object. Such branching solutions were mostly found in indeterminate problem cases (14 out of the 17 cases in total). We had to remove these cases from the final data set as we were not able to extract a single unambiguous solution. However, these branching solutions clearly reflect a special type of errors, as they usually reflected the indeterminate nature of the problems. Note, however, that by removing these trials from the analysis, we artificially increased performance primarily for in- determinate problems, which might explain the surprisingly high performance in these problems.

An analysis of the chosen solutions for indeterminate problems clearly demonstrated that participants did not choose randomly between the three possible solutions, but preferred one over the others. The preferred solution was identical to the one generated by the first-free fit strategy, a preferred mental model generation strategy identified in previous experiments (Ragni et al., 2007) on small-scale scenarios. Then again, the constraints in this experiment were clearly spatio-temporal in their nature – the premises “the fountain of Trevi before the colosseum” refers to the sequence of the events. In that sense, it is not surprising that the identified preferences were similar to those identified in small-scale scenarios (Schaeken, Johnson Laird, & d’Ydewalle, 1996).

General Discussion

In dealing with maps there is one important and new question: What is the influence of the (implicit) task of planning a short path using maps while taking into account spatio-temporal constraints? The way reasoners typically construct preferred mental models when reasoning about indeterminate problems has been identified in several experiments (cp. Ragni et al., 2007). The most prominent encoding strategy applied in such cases is the first-free- fit strategy. This strategy, however, does not al- low for predicting how external constraints such as the length of the routes resulting from reasoning problems influence the reasoning process itself. In this study we combined reasoning about spatio-temporal constraints with the task of planning short paths with a map (without explicitly stating that the shortest path must be found). The planning task influenced the reasoning task: In de- terminate problems – in which only a single solution existed – participants showed better performance if that solution was identical to the shortest possible path. Furthermore, for indeterminate cases, we found strong preferences for the solution that corresponded to the first-free-fit strategy. Participants’ performance in finding a possible solution for IP-suboptimal problems was greater than for IP- optimal problems. This result is surprising and was not predicted, as the optimal solution to the route-planning problem – i.e., the shortest possible route – was identical to the preferred mental model for IP- optimal problems but not for IP-suboptimal problems. Future research will address this interesting effect.

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