Mathematically Modeling Anchoring Effects

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Abstract

This article proposes a method by which anchoring effects can be mathematically modeled. Anchoring effects are a type of assimilation effect; so this article proposes using Anderson's (1965; 1981) integration model to model anchoring effects, as it is typically used to model other assimilation effects. The difficulty in using the integration model is that doing so requires that the modeler knows or is able to estimate participants' unbiased estimates (i.e., what their estimates would have been had they never seen the anchor) and this information is not available from conventional anchoring effect paradigms. A method for estimating unbiased estimates is proposed. This method is used to estimate unbiased estimates for a set of anchoring effect data and the integration model is fit to these data. This article closes by speculating on possible theoretical insights into anchoring effects that might be gleaned by using the proposed methodology and possible practical applications.

Anchoring Effects

The goal of this paper is to propose a method by which anchoring effects can be mathematically modeled. The ability to mathematically model anchoring effects might be useful both for differentiating among theoretical models of anchoring effects and for calculating likely practical applications of anchoring effects in situations such as negotiations (e.g., Chapman & Bornstein, 1996; Galinsky & Mussweiler, 2001), auctions (Ku, Galinsky, & Murnighan, 2006), and pricing (Northcraft & Neale, 1987). These possible applications of the proposed model will be discussed in the General Discussion section.

In anchoring effects, estimates of an unknown value are assimilated towards an arbitrary numeric value called the anchor. For example, in a well-known study, Tversky and Kahneman (1974) asked participants to judge whether African nations represented a higher or lower percentage of UN-member nations than an anchor and then to estimate the actual percentage. Estimates were assimilated towards the anchor. When the anchor was 10% of UN-member nations, the median estimate was assimilated downward toward 10% to equal 25%; but when the anchor was 65%, the median estimate was assimilated upward toward 65% to equal 45%.

Assimilation effects like these are typically mathematically modeled using Anderson's (1965; 1981) integration model. A mathematical formalization like the integration model formalization was alluded to in at least one anchoring effect paper (see Jacowitz & Kahneman, 's, 1995, discussion of priming models of anchoring effects). In addition, this mathematical formalization has been used to model assimilation effects in phenomena as diverse as impression formation (the domain that originally inspired Anderson's model, see Urada, Stenstrom, & Miller, 2007, for a recent application), physical attractiveness (e.g., Wedell, Parducci, & Geiselman, 1987), product evaluation (e.g., Miyazaki, Grewal, & Goodstein, 2005; Troutman & Shanteau, 1976), risk assessment (e.g., Hampson, Andrews, Barckley, Lee, & Lichtenstein, 2003), and the best timing for lesbian and gay politicians to come out of the closet (Golebiowska, 2003)[1].

The Proposed Mathematical Model

Anderson's (1965; 1981) integration model would model the assimilation observed in anchoring effects as a weighted average of the anchor value (A) and the unbiased estimate a participant would have made had she or he never seen the anchor (U: U for Unbiased; see below for how this quantity can be empirically measured):

$$EST = wA + (1-w)U \qquad (1)$$

where EST represents a participant's estimate (i.e., the dependent measure in anchoring estimation tasks) and w is the weight bound between 0 and 1 of the anchor value (A) relative to the unbiased estimate (U). A weight of 0 would represent a case in which estimates were not affected at all by exposure to the anchor. In such a case, unbiased estimates (U) would be equal to participants' estimates (EST) so that EST = U. A weight of 1 would represent a case in which all participants simply respond with the anchor value. Weights between these two extremes represent estimates that are assimilated toward the anchor value, but are not equal to it.

The key problem in using Anderson's (1965; 1981) integration model to model anchoring effects is that it requires the modeler to know what participants' unbiased estimates (U) would have been had they never seen the anchor. Measuring these unbiased estimates is made particularly difficult, because it is not possible to ask participants to make a numerical estimate twice (once before and once after being exposed to the anchor value) as their first numerical estimate will bias their second. To solve this problem, the methodology proposed here would have participants make a non-numerical estimate before being exposed to the anchor and then make a numerical estimate afterwards. The mapping between non-numerical estimates and numerical estimates can then be established by running a control condition in which participants make both types of estimates without being exposed to the anchor and calculating a regression line between the two types of estimates. The unbiased estimates (U) of the participants in the experimental condition can then be calculated using the non-numerical estimates that these participants make and the regression line.

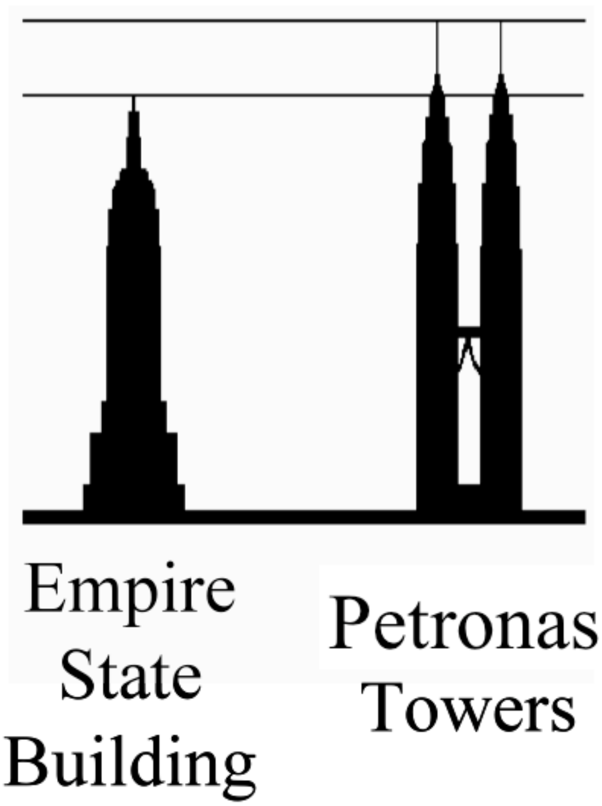

Figure 1. Graphic estimate of the height of the Sears Tower. Participants placed a tick mark between the horizontal line representing the height of the Empire State Building and the horizontal line representing the height of the Petronas Towers to represent how tall they believed the Sears Tower to be.

In the data modeled below, for example, the task was to estimate the height of the Sears Tower (a Chicago landmark and one of the world's tallest buildings; since the time during which these data were collected, this building has been renamed the Willis Tower). Participants made two estimates: a non-numerical estimate and a numerical estimate. The non-numerical estimate was made on the graphic presented in Figure 1. Participants were told that the Empire State Building was the tallest building in the world until the Sears' Tower was built and that the Sears' Tower was the tallest building in the world until the Petronas Towers in Kuala Lumpur, Malaysia were built (taller buildings yet have been built since the Petronas Towers were built). Participants made a tick mark between the two horizontal lines in Figure 1 to denote how tall they believed the Sears Tower to be relative to the Empire State Building and the Petronas Towers. The distance between the bottom line representing the height of the Empire State Building and each participant's tick mark was then measured in millimeters (mm).

The numerical estimate was the number of feet tall that participants estimated the Sears Tower to be. Participants in the control condition made the non-numerical estimate and then the numerical estimate without being exposed to the anchor. Participants in the experimental condition made the non-numerical estimate before they made a judgment regarding whether the Sears Tower was taller or shorter than the anchor value of 1,367 feet and then made the numerical estimate. A regression line was calculated between the control participants' non-numerical and numerical estimates. This regression line was then used to calculate the experimental participants' unbiased estimates (U) from their non-numerical estimates.

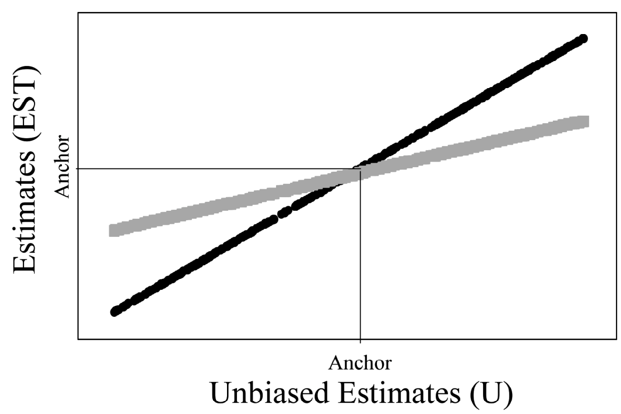

Figure 2. Anchoring effect that would be characterized as an assimilation effect. The black line represents the predicted pattern of estimates, if estimates were not affected by the anchor. The gray line represents the predicted pattern of estimates, if an assimilation effect were observed. Notice that the gray line represents a weighted average of the black line (estimates unbiased by the anchor) and the anchor value (See Equation 1).

A pattern of biases that would fit Anderson's (1965; 1981) integration model definition of an assimilation effect as presented in Equation 1 is demonstrated in Figure 2. The x-axis represents unbiased estimates (U) and the y-axis represents participants' estimates in anchoring estimation tasks (EST). Do not confuse this figure with the similar-looking figures used by Chapman and Johnson (1994). In Chapman and Johnson's figures, the x-axis represented alternative anchor values. In Figure 2, the x-axis represents unbiased estimates and the location labeled “Anchor” represents a situation wherein a participant’s unbiased estimate just happened to be equal to the anchor value. The black line in Figure 2 represents what the pattern of estimates would look like, if the anchor did not bias estimates (i.e., the case in which w = 0 and EST = U). The gray line represents a pattern of biased estimation that would be characterized as an assimilation effect (any linear slope between the slope of the black line and horizontal—that is, where w in Equation 1 takes a value greater than zero and less than one—would be classified as an assimilation effect).

Notice that regardless of the values of the unbiased estimates (U), Equation 1 predicts that they will be biased toward the anchor value by the same proportion. For example, all values might be biased 20% toward the anchor. Sometimes the term “assimilation effect” has been used roughly to refer to any bias towards a standard regardless of the extent of the bias and whether the bias toward the standard is uniform (e.g., Schwarz & Bless, 1992). While using the term in this way often provides a useful way to quickly classify results (i.e., as either “assimilation,” bias toward or “contrast,” bias away from a standard), Anderson’s (1965; 1981) definition is more precise in that it captures the degree of bias toward the anchor across the entire range of unbiased estimates and provides a starting point from which to model anchoring effects. If it turns out that not all estimates are biased toward the anchor by the same proportion (e.g., unbiased estimates close to the anchor might be biased towards the anchor by a smaller proportion than unbiased estimates that are farther away from the anchor or vice versa), then the methodology proposed here can also be used to fit alternative equations—other than the integration theory equation—to anchoring effect data.

We used this methodology and collected anchoring effect data to which Anderson’s (1965; 1981) integration model could be fit.

Anchoring Effect Data

The purpose of the experiment reported here was to use the methodology proposed above to collect data to which mathematical models—Anderson’s (1965; 1981) integration model, in particular—could be fit. There was an experimental group of participants and a control group. The experimental group made a non-numerical estimate of the height of the Sears’ Tower, then compared its height to the anchor value of 1,367 feet, and finally made a numerical estimate of the height of the Sears’ Tower in feet. The control group made a non-numerical estimate and then a numerical estimate without ever being exposed to the anchor.

Method

Participants. One hundred sixty passengers on the Chicago elevated train system participated voluntarily (80 in the control condition and 80 in the experimental condition).

Materials and Procedure. We told our participants that the Empire State Building was the tallest building in the world until the Sears Tower was built and that the Sears Tower was the tallest building in the world until the Petronas Towers were built. To measure unbiased estimates, we first asked participants to estimate the height of the Sears Tower graphically by showing them in-scale silhouettes of the Empire State Building and the Petronas Towers as shown in Figure 1. Horizontal lines crossed the page to represent the height of each skyscraper. Participants placed a tick mark between the lines to represent their estimates of the height of the Sears Tower. After estimating the height of the Sears Tower graphically, participants in the control condition simply estimated the height of the Sears Tower in feet (numerical estimate). Participants in the experimental condition judged the height of the Sears Tower to be “more” than or “less” than the anchor value of 1,367 feet before estimating the height of the Sears Tower in feet (numerical estimate).

Results

The results are presented in Figure 3. As noted in the discussion of Figure 2 above, be careful not to confuse these figures with the similar-looking figures used by Chapman and Johnson (1994). The x-axis here represents unbiased estimates as measured using the graphic presented in Figure 1; and the y-axis represents participants’ numerical estimates in feet. We first investigated whether an anchoring effect was observed by performing a t-test on the absolute difference between participants’ numerical estimates in feet and the anchor value of 1,367 feet. The anchoring effect was highly reliable, $t(158)=4.72$, $p<.01$. Estimates were significantly closer to the anchor value in the experimental condition (M=128.30 feet away from 1,367 feet, SD=127.93) than in the control condition (M=479.90 feet away from 1,367 feet, SD=654.42).

Fitting the Model

Equation 1 was fit to the results of this experiment. The criterion variable, EST, represented each participant’s estimate. To use Equations 1 to predict EST, one must somehow measure the estimates participants would have made had they never seen the anchor value. That is, one must measure participants’ unbiased estimates, Parameter U. To do so, we used the results from the control group to regress their non-numerical estimates (as collected using the graphic presented in Figure 1 and measured on mm from the bottom horizontal line representing the height of the Empire State Building) on their numerical estimates. We then used this regression equation along with each experimental participant’s non-numerical estimate to predict what their unbiased numerical estimates, U, would have been had they never seen the anchor. The regression line predicts U as: U=766.12+(50.93*the distance in mm between the bottom line in Figure 1 representing the height of the Empire State Building and each participant’s tick mark). With EST equal to each experimental participant’s estimate and U equal to the value predicted by the regression equation, assimilation effects toward the anchor were modeled using Equation 1.

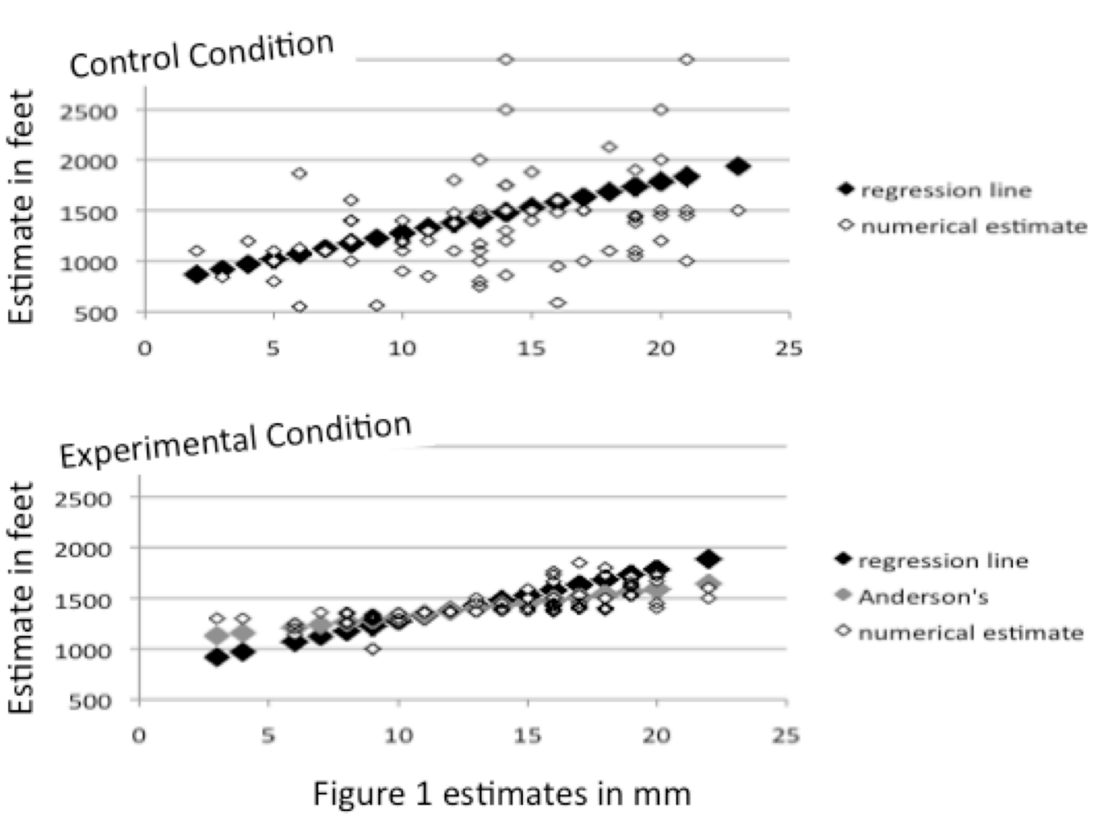

Figure 3. Results of the anchoring effect experiment reported here including the regression line and Anderson's (1965;1981) integration model fits. The x-axis represents participants' unbiased estimates of the height of the Sears/Willis Tower on the graphic presented in Figure 1 and the y-axis represents participants' numerical estimates of the height in feet. The white diamonds represent particular participants' estimates; the black diamonds represent the regression line calculated on the control participants' estimates; and the gray diamonds represent the best fit from Anderson's integration model.

Parameter A, representing the anchor value, took a value of 1,367 feet and the best-fitting value for Parameter w was calculated using a root mean squared error (RMSE) criterion. The best-fitting value for Parameter w was 0.47; and the RMSE was 116.93. A paired sample t-test on the squared errors of the values predicted by Anderson's integration model versus the squared errors of the values predicted by the regression equation found that Anderson's integration model provided a better fit, $t(79)=3.81$, $p<.01$.

Discussion

A method of mathematically modeling anchoring effects was proposed. This method calculated unbiased estimates (the estimates participants would have made had they never seen the anchor value) by having participants make a non-numerical estimate before being exposed to the anchor value and a numerical estimate afterwards. The mapping between non-numerical estimates and numerical estimates was calculated by asking a control group of participants to make both types of estimates without ever being exposed to the anchor and calculated a regression line between the two types of estimates. The regression line along with the non-numerical estimates of the experimental participants allowed us to estimate what the experimental participants' estimates would have been had they never been exposed to the anchor value. Anderson's (1965; 1981) integration model (Equation 1) was then fit to these data where U represented each experimental participants' unbiased estimate as calculated by the regression line, EST represented each participants' numerical estimate, and A represented the anchor value of 1,367 feet. The best fitting value for parameter w using a RMSE criterion was 0.47.

Future research should fit other types of equations to anchoring effect data collected using this method. Doing so might prove useful for further refining theoretical models of anchoring effects. For example, if the anchor value is outside of the range of acceptable estimates, then Tversky and Kahneman's (1974) account of anchoring effects—under which anchors provide a starting point for participants' search for an appropriate estimate—would not produce a pattern of results that should be modeled using Anderson's integration model. Instead of predicting that all unbiased estimates would be biased toward the anchor by the same proportion, Tversky and Kahneman's (1974) account would predict an approximately horizontal estimation function. It would predict a horizontal estimation function, because all participants would start their search for an appropriate value at the anchor value which is outside of the range of acceptable estimates, and adjust from there, stopping at the first acceptable value. They would do so regardless of what their unbiased estimates would have been had they never been exposed to the anchor value. One qualification on this prediction of the anchoring and adjustment model of anchoring effects would be if the range of values that participants thought acceptable correlated with their unbiased estimates, but this question could be addressed in future research as well (by having control participants identify the range of values they consider acceptable) and the issue would not have been addressable without the methodology proposed here.

By contrast, priming models of anchoring effects (Wilson, et al., 1996; Wong & Kwong, 2000) would predict estimation functions that would follow Anderson's integration model pattern (see Jacowitz & Kahneman, 's, 1995, discussion of priming models of anchoring effects). Exposure to the anchor value would prime that value and then estimates would be a weighted average between the primed values and the unbiased estimates participants would have made had they never been exposed to the anchor.

The pattern of bias predicted by Mussweiler and Strack's (1999; see also Strack & Mussweiler, 1997) selective accessibility model is less clear. The selective accessibility model assumes that when people compare the unknown, to-be-estimated value to the anchor value, they test whether the unknown, to-be-estimated value might be the same as the anchor value by searching for semantic information that would confirm that the to-be-estimated value is equal to the anchor value. Confirmation biases almost always produce a situation wherein people are able to find semantic information about the to-be-estimated value suggesting that it is equal to the anchor value. If this account of anchoring effects is correct, then the degree of bias toward the anchor will depend upon the amount of confirmatory information they are able to recall. The ability to find such confirmatory evidence may vary as a function of people's unbiased estimates. People whose estimates would have otherwise suggested a value close to the anchor based upon their unbiased semantic knowledge of the to-be-estimated value may be more likely to find confirmatory evidence than people whose unbiased estimates would have otherwise been farther away. The proportion of bias towards the anchor may then be greater for unbiased estimates that are relatively close to the anchor than for unbiased estimates that are farther away from the anchor. Furthermore, future work might investigate the role of selective accessibility mechanisms in anchoring effects by using the methodology proposed here to investigate anchoring effects when participants have a great deal of semantic knowledge about the to-be-estimated value and when they do not.

The methodology proposed here (perhaps using a rating scale to measure unbiased estimates, rather than the measure presented in Figure 1) may also be useful for studying practical applications of anchoring effects in situations such as negotiations (e.g., Chapman & Bornstein, 1996; Galinsky & Mussweiler, 2001), auctions (Ku, Galinsky, & Murnighan, 2006), and pricing (Northcraft & Neale, 1987). For example, starting negotiations over the selling price of a home at a high initial asking price may have different effects depending upon what the potential buyer's unbiased estimate of a reasonable price for the house would have been had she or he never heard the asking price. It is not clear *a priori* whether all buyers' bids are biased toward the initial asking price by the same proportion. It might turn out that closer unbiased estimates are biased toward the initial asking price by a smaller proportion; or it might turn out that they are biased toward the initial asking price by a greater proportion. If it turns out that closer unbiased estimates are biased toward the initial asking price by a greater proportion, then it may not be the case that larger initial asking prices always produce the highest selling prices even if on average they do so. It may turn out that this phenomenon is mostly due to people who's unbiased estimates would have been relatively high before hand and the bias just makes their estimates of an appropriate bid higher yet. If so, then lower initial asking prices might be more effective in producing high selling prices among the segment of consumers whose unbiased estimates of an appropriate price were not quite as high at the start. If so, then the methodology proposed here might be useful in setting optimal initial asking prices for the entire range of potential consumers.

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